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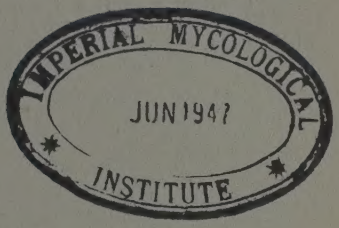

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MATHEMATICS

ON A GENERALIZATION OF SOME INVESTIGATIONS OF G. SZEGÖ, V. SMIRNOFF AND A. KOLMOGOROFF

By M. KREIN

(Communicated by A. N. Kolmogoroff, Member of the Academy, 18.VIII.1944)

Generalizing in the present paper the G. Szegö ([2], [3]) and V. Smirnoff ([4]) theory of orthogonal polynomials on a contour, as well as some of the results of A. Kolmogoroff ([1]), whick make this theory in a certain sense complete, we confine ourselves for want of space to the case of functions defined on the unit circle. The whole theory may be, however, developed also in the case of functions defined on an arbitrary Jordan contour (which need not be rectifiable).

1. In the sequel L denotes a linear aggregate of functions $f(e^{i\vartheta})$ $(0 \leqslant \vartheta \leqslant 2\pi)$ possessing the following properties:

1) If $f(e^{i\vartheta}) \in L$, then $e^{i\vartheta} f(e^{i\vartheta}) \in L$.

2) $1 \in L$.

If $\sigma(\vartheta) = \sigma(\vartheta - 0)$ $(0 \leqslant \vartheta \leqslant 2\pi,\ \sigma(0) = 0)$ is a non-decreasing function, then by L_σ we denote the corresponding Hilbert space of all σ-measurable and σ-integrable together with their squares functions $\varphi(\vartheta)$ $(0 \leqslant \vartheta \leqslant 2\pi)$ with the usual definition of the scalar product:

$$(\varphi, \psi)_\sigma = \int_0^{2\pi} \varphi(\vartheta)\, \overline{\psi(\vartheta)}\, d\sigma(\vartheta) \qquad (\varphi,\ \psi \in L_\sigma)$$

By V_L we denote the aggregate of all non-decreasing functions $\sigma(\vartheta) = \sigma(\vartheta - 0)$ $(\sigma(0) = 0)$ for which $L \subset L_\sigma$.

Finally, by H_ϑ we denote the aggregate of all measurable and summable with their squares functions $F(e^{i\vartheta})$ $(0 \leqslant \vartheta \leqslant 2\pi)$, the Fourier coefficients of which

$$c_n(F) = \frac{1}{2\pi} \int_0^{2\pi} F(e^{i\vartheta})\, e^{-ni\vartheta}\, d\vartheta$$

are equal to zero for $n < 0$. If $F(e^{i\vartheta}) \in H_\vartheta$, then by $F(z)$ $(|z| < 1)$ we shall denote the corresponding holomorphic function

$$F(z) = \sum_{n=0}^{\infty} c_n(F)\, z^n \qquad (|z| < 1)$$

The aggregate of all holomorphic functions $F(z)$ $(|z| < 1)$ corresponding to the functions $F(e^{i\vartheta}) \in H_\vartheta$ we shall denote by H_z.

Theorem 1. *Let $\sigma \in V_L$. Then L is not dense in L_σ if and only if the following two conditions are satisfied:*

$$1^\circ \qquad -\infty < \int_0^{2\pi} \log \sigma'(\vartheta)\, d\vartheta \qquad (1)$$

2° *There exists a function $F(e^{i\vartheta}) \in H_\vartheta$ such that*

$$\int_0^{2\pi} \frac{|F(e^{i\vartheta})|^2}{\sigma'(\vartheta)}\, d\vartheta < \infty$$

and $f(e^{i\vartheta}) F(e^{i\vartheta}) \in H_\vartheta$ for any function $f \in L$.

Proof. Consider in L_σ the unitary operator U determined by the equality: $U\varphi = e^{i\vartheta}\varphi(\vartheta)$ $(\varphi \in L_\sigma)$. Denote by $\overline{L}$ the closure of L in L_σ. Since $UL = L$, we have also, in virtue of the continuity of the operator U, $U\overline{L} \subset \overline{L}$. If $\overline{L} \subset L_\sigma$, then $U\overline{L} = \overline{L}$. Conversely, if $U\overline{L} = \overline{L}$, then $U^k\overline{L} = \overline{L}$ $(k = 0, \pm 1, \pm 2, \ldots)$, and since $1 \in L$, $e^{ki\vartheta} = U^k 1 \in \overline{L}$ $(k = 0, \pm 1, \pm 2, \ldots)$, whence follows $\overline{L} = L_\sigma$. Thus, the fact that L is not dense in L_σ is equivalent to the fact that $\overline{UL} \neq \overline{L}$.

But if $U\overline{L} \neq \overline{L}$ $(UL \in L)$, there is a function $\Phi(e^{i\vartheta}) \in \overline{L}$ such that

$$(\Phi, \Phi)_\sigma = 1, \quad (\Phi, Uf)_\sigma = 0 \text{ for } f \in L$$

In particular, we shall have $(\Phi, U^k\Phi)_\sigma = 0$ $(k = 0, 1, 2, \ldots)$; thus

$$\int_0^{2\pi} |\Phi(e^{i\vartheta})|^2\, d\sigma(\vartheta) = 1, \quad \int_0^{2\pi} e^{ki\vartheta} |\Phi(e^{i\vartheta})|^2\, d\sigma(\vartheta) = 0 \quad (k = \pm 1, \pm 2, \ldots)$$

But this is possible only in the case where

$$\int_0^{\vartheta} |\Phi(e^{i\vartheta})|^2\, d\sigma(\vartheta) = \frac{\vartheta}{2\pi} \qquad (0 \leqslant \vartheta \leqslant 2\pi)$$

Hence follows that p.p.

$$\sigma'(\vartheta) = \frac{1}{2\pi} |F_\sigma(e^{i\vartheta})|^2, \quad F_\sigma(e^{i\vartheta}) = \Phi^{-1}(e^{i\vartheta}) \qquad (2)$$

and

$$\int_0^{\vartheta} |\Phi(e^{i\vartheta})|^2\, d\sigma_s(\vartheta) = 0,$$

where

$$\sigma(\vartheta) = \int_0^{\vartheta} \sigma'(\vartheta)\, d\vartheta + \sigma_s(\vartheta) \qquad (0 \leqslant \vartheta \leqslant 2\pi) \qquad (3)$$

We apply now the fact that for any function $f \in \overline{L}$ we have $(\Phi, U^k f)_\sigma = 0$ $(k = 1, 2, \ldots)$, *i. e.*

$$\int_0^{2\pi} e^{ki\vartheta} f(e^{i\vartheta}) \overline{\Phi(e^{i\vartheta})}\, d\sigma(\vartheta) = \frac{1}{2\pi} \int_0^{2\pi} e^{ki\vartheta} f(e^{i\vartheta}) F_\sigma(e^{i\vartheta})\, d\vartheta = 0 \qquad (k = 1, 2, \ldots)$$

Thus

$$F_\sigma(e^{i\vartheta}) f(e^{i\vartheta}) \in H_\vartheta \text{ for any } f \in \overline{L} \qquad (4)$$

Putting in particular $f \equiv 1 \in L$, we find that $F_\sigma(e^{i\vartheta}) \in H_\vartheta$ and this in view of (2) implies 1° (cf. [5], ch. IV).

To prove the sufficiency of conditions 1°, 2° let us put, starting from the function F given by 2°

$$G(\theta) = e^{-i\theta} F(e^{i\theta}) / \sigma'(\theta)$$

for those θ for which there exists a finite derivative $\sigma'(\theta) \neq 0$; for other θ we put $G(\theta) = 0$. Then

$$\int_0^{2\pi} |G(\theta)|^2 \, d\sigma(\theta) > 0, \quad \int_0^{2\pi} f(e^{i\theta}) \overline{G(\theta)} \, d\sigma(\theta) = \int_0^{2\pi} f(e^{i\theta}) \overline{F(e^{i\theta})} \, e^{i\theta} \, d\theta = 0 \qquad (f \in L)$$

i. e. $\overline{L} \neq L_\sigma$. The theorem is proved.

2. If the condition 2° of Theorem 1 is satisfied, we have:

3) To every function $f(e^{i\theta}) \in L$ there corresponds a meromorphic function $f(z)$ $(|z| < 1)$ of bounded type such that p.p. $f(re^{i\theta}) \to f(e^{i\theta})$ for $r \to 1$ *.

In fact, if $F(e^{i\theta}) \in H_\theta$ and $G(e^{i\theta}) = f(e^{i\theta}) F(e^{i\theta}) \in H_\theta$ $(f \in L)$, we may put $f(z) = G(z) : H(z)$ $(|z| < 1)$, and since the holomorphic functions $G(z) \in H_z$, $F(z) \in H_z$ are of bounded type (cf. [5], ch. IV, VI), the function $f(z)$ will be of the same type.

Theorem 2. *If $\sigma \in V_L$ and L is not dense in L_σ, then there exists a convergent Blaschke product*

$$B(z) = z^p \prod \frac{a_k - z}{1 - \bar{a}_k z} \frac{|a_k|}{a_k} \qquad (p \geqslant 0, \ 0 < |a_k| < 1, \ k = 1, 2, \ldots).$$

possessing the property that every function $B(z) f(z)$ $(f \in L)$ is holomorphic for $|z| < 1$. If we choose the function $B(z)$ in such a way that it should not have «superfluous» zeros, then the function

$$F_\sigma(z) = \sqrt{2\pi} \, B(z) \exp\left\{ \frac{1}{4\pi} \int_0^{2\pi} \frac{e^{i\theta} + z}{e^{i\theta} - z} \log \sigma'(\theta) \, d\theta \right\} \quad (|z| < 1) \qquad (5)$$

will possess the following properties:

a) *$F_\sigma(z) \in H_z$ and, moreover, $F_\sigma(z) f(z) \in H_z$ for every $f \in L$;*

b) *p.p. $\sigma'(\theta) = 1/2\pi \, |F_\sigma(e^{i\theta})|^2$ where $F_\sigma(e^{i\theta}) = \lim\limits_{r \uparrow 1} F_\sigma(re^{i\theta})$;*

c) *if we put $\Phi_\sigma(e^{i\theta}) = F_\sigma^{-1}(e^{i\theta})$ at points where $F_\sigma(e^{i\theta}) \neq 0$ and where there exists the finite derivative $\sigma'(\theta)$, and put $\Phi_\sigma(e^{i\theta}) = 0$ at other points, then $\Phi_\sigma(e^{i\theta}) \in \overline{L}$ ($\overline{L}$ being the closure of L in L_σ);*

d) *whatever the orthonormed system $\{D_k\} \subset L$ complete in L, we will have for any two points z and ζ $(|z| < 1, \ |\zeta| < 1)$ different from the zeros of $B(z)$*

$$(1 - z\bar{\zeta}) \sum_{k=1}^{\infty} D_k(z) \overline{D_k(\zeta)} = F_\sigma^{-1}(z) \overline{F_\sigma^{-1}(\zeta)} \qquad (6)$$

For want of space we omit the proof of this theorem.

Now the following theorem may be easily proved.

Theorem 3. *Under the conditions of Theorem 2 the following assertions are equivalent:*

α) *the meromorphic function $g(z)$ $(|z| < 1)$ is developable into the series*

$$g(z) = \sum_1^\infty c_k D_k(z), \quad \sum_1^\infty |c_k|^2 < \infty \quad (|z| < 1, \ B(z) \neq 0) \qquad (7)$$

β) *the meromorphic function $g(z)$ $(|z| < 1)$ satisfies the condition $F_\sigma(z) \, g(z) \in H_z$;*

---

* A meromorphic function $g(z)$ $(|z| < 1)$ is called a function of bounded type if it is representable as a ratio of two bounded holomorphic functions; such a function is uniquely determined by its boundary values $g(e^{i\theta})$ existing for almost all θ (cf. [5], ch. VI).

γ) *the meromorphic function* $g(z)$ $(|z|<1)$ *is of bounded type, and its boundary function* $g(e^{i\vartheta})$ *coincides almost everywhere with a certain function* $g_0(e^{i\vartheta}) \in \overline{L}$.

If (7) *holds, then*

$$\int_0^{2\pi} |g_0(e^{i\vartheta})|^2 \, d\sigma(\vartheta) = \sum_1^{\infty} |c_k|^2, \quad c_k = \int_0^{2\pi} g_0(e^{i\vartheta}) \overline{D_k(e^{i\vartheta})} \, d\sigma(\vartheta) \quad (k = 1, 2, \ldots) \tag{8}$$

where $g_0(e^{i\vartheta}) \in \overline{L}$ *almost everywhere coincides with* $g(e^{i\vartheta})$.

Corollary 1. In the case where L is the set of all polynomials of e^{i}, in Theorem 1 we must omit the condition 2° (it follows from the condition 1°), and in Theorem 2 we have to put $B(z) \equiv 1$. For this case Theorem 1 and the fundamental property c) from Theorem 2 of the function $F_\sigma(z)$ were established by A. Kolmogoroff [1] by means of the notions and arguments of the theory of stationary sequences.

Corollary 2. The orthonormed sequence $\{D_k\} \subset L$ may be constructed in such a way that the function $\zeta D_k(\zeta)$ $(k = 1, 2, \ldots, \zeta = e^{i\vartheta})$ will be linearly expressible through a finite number of the same functions $D_j(\zeta)$ $(j = 1, 2, \ldots)$.

In fact, let $\{\chi_j\}$ be a certain system of functions from L whose linear envelope is dense in L. Let us denumerate the functions

$$\zeta^k \chi_j(\zeta) \quad (j, k = 1, 2, \ldots)$$

omitting those of them which linearly depend on the foregoing ones. Orthonorming the sequence thus obtained by E. Schmidt's process, we arrive at the required sequence $\{D_k\}$.

If L possesses the properties 1), 2), 3) and the orthonormed sequence $\{D_k\}$ satisfies the condition mentioned above, then we may assert that *either the series*

$$p(z) = \sum_1^{\infty} |D_k(z)|^2 \quad (|z|<1) \tag{9}$$

diverges everywhere and L *is dense in* L_σ, *or* L *is not dense in* L_σ *and the series* (9) (*by Theorem* 2) *converges at all points* z $(|z|<1)$ *where* $B(z) \neq 0$.

Corollary 3. If L is the set of all polynomials of $e^{i\vartheta}$, and $\sigma(\vartheta)$ is an absolutely continuous function satisfying the condition (1), then the formulations of Theorems 2 and 3 may be simplified in an obvious manner. In this case the Theorems 2 and 3 were established by G. Szegö [2] and V. Smirnoff [4] [cf. also [3]].

Concluding we may observe that the Theorems 1, 2 and 3 hang closely together with Theorem 8 of our paper [6].

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REFERENCES

[1] А. Н. Колмогоров, Бюллетень МГУ, Математика, II, в. 6 (1941); Bull. Acad. Sci. URSS, sér. math., 5, № 1 (1943). [2] G. Szegö, Math. Z., 6, H. 3/4 (1920); 9, H. 3/4 (1921). [3] G. Szegö, Orthogonal polynomials, 12, 1, N. Y. (1939). [4] В. И. Смирнов, Журн. Ленинградск. физ.-математич. об-ва, II, в. 1, 155; Bull. Acad. Sci. URSS, sér. math. et sci. nat., VII, № 3 (1932). [5] И. И. Привалов, Граничные свойства однозначных аналитических функций, 1941. [6] M. Krein, C. R. Acad. Sci. URSS, XLIV, No. 6 (1944).

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MATHÉMATIQUES

SUR UN NOUVEAU CAS D'INTÉGRABILITÉ DANS LE PROBLÈME RECTILIGNE DES TROIS CORPS

Par G. SOKOLOFF

(Présenté par S. L. Soboleff, de l'Académie, le 6.IV.1944)

Considérons le mouvement plan des trois points matériels P_0, P_1, P_2 de masses m_0, m_1, m_2 qui s'attirent [si $f(r_k) < 0$] ou se repoussent [si $f(r_k) > 0$] mutuellement, la force qui s'exerce entre ces points étant

$$m_i m_j |f(r_k)| \qquad (i, j, k = 0, 1, 2; \quad i \neq j \neq k)$$

où $r_k = \overline{P_i P_j}$, et $f(r) = dF(r)/dr$ est une fonction quelconque de la distance.

En étudiant le mouvement relatif du point P_1 par rapport à P_0 et du point P_2 par rapport au centre d'inertie des masses m_0 et m_1, désignons par $r = r_2$, ϑ, ρ, ω les coordonnées polaires de ces points et prenons pour inconnues du problème les fonctions, définies par les formules

$$I\cos\chi = \sqrt{\mu_2}\, r, \quad I\sin\chi = \sqrt{\mu_2}\,\rho \left(\mu_2 = \frac{m_0 m_1}{m_0 + m_1}, \quad \mu_3 = \frac{m_2(m_0+m_1)}{m_0+m_1+m_2}, \quad 0 < \chi < \frac{\pi}{2}\right)$$

$$R = \mu_2 r r' + \mu_2 \rho\rho', \qquad X = \sqrt{\mu_2 \mu_3}\,(r\rho' - \rho r')$$

$$\Theta = \mu_2 r^2 \vartheta', \qquad \Omega = \mu_3 \rho^2 \omega'$$

En prenant I pour la variable indépendante et en utilisant les intégrales des forces vives et des aires on peut ramener le problème à l'intégration d'un système de quatre équations du premier ordre:

$$\left.\begin{aligned} I\frac{d\chi}{dI} &= \frac{X}{R}, \quad I\frac{dX}{dI} = \frac{I^2}{R}\frac{\partial U}{\partial \chi} + \frac{\sin 2\chi}{2R}\left(\frac{\Omega^2}{\sin^4\chi} - \frac{\Theta^2}{\cos^4\chi}\right) \\ I\frac{d\tau}{dI} &= \frac{1}{R}\left(\frac{\Theta}{\cos^2\chi} - \frac{\Omega}{\sin^2\chi}\right), \quad \frac{d\Theta}{dI} = \frac{I}{R}\frac{\partial U}{\partial \tau} \end{aligned}\right\} \qquad (A)$$

où

$$R = \sqrt{2I^2 U + 2hI^2 - X^2 - \frac{\Theta^2}{\cos^2\chi} - \frac{\Omega^2}{\sin^2\chi}},$$

$$\Omega = c - \Theta, \quad \tau = \vartheta - \omega$$

(h et c sont des constantes des intégrales des forces vives et des aires),

$$U = \sum m_i m_j F(r_k), \quad \frac{\partial U}{\partial \chi} = \sum m_i m_j f(r_k)\frac{\partial r_k}{\partial \chi},$$

$$\frac{\partial U}{\partial \tau} = \frac{m_2}{2}\sqrt{\frac{\mu_2}{\mu_3}}\, I^2 \sin 2\chi \sin\tau \left[\frac{f(r_0)}{r_0} - \frac{f(r_1)}{r_1}\right]$$

$$r = \frac{I\cos\chi}{\sqrt{\mu_2}}, \quad r_0 = \frac{I}{\sqrt{\mu_2\mu_3}}\sqrt{\mu_3 \sin^2\chi + \mu_0^2\mu_3\cos^2\chi - \mu_0\sqrt{\mu_2\mu_3}\sin 2\chi\cos\tau}$$

$$r_1 = \frac{I}{\sqrt{\mu_2\mu_3}} \sqrt{\mu_2 \sin^2\chi + \mu_1^2\mu_3 \cos^2\chi + \mu_1\sqrt{\mu_2\mu_3}\sin 2\chi \cos\tau}$$

$$\left(\mu_0 = \frac{m_0}{m_0+m_1}, \quad \mu_1 = \frac{m_1}{m_0+m_1}\right)$$

et à deux quadratures

$$\vartheta = \vartheta_0 + \int_{I_0}^{I} \frac{\theta dI}{IR\cos^2\chi}, \qquad t = t_0 + \int_{I_0}^{I} \frac{IdI}{R}$$

Par la considération du système (A) il est aisé d'obtenir tous les cas classiques d'intégrabilité à l'aide des quadratures, ainsi que leur certaines généralisations.

1. Cas équidistant

$$\chi = \chi_0 = \operatorname{arc\,tg}\sqrt{\frac{\mu_3}{\mu_2}(1-\mu_0\mu_1)}, \quad \tau = \tau_0 = \arccos\frac{\mu_0-\mu_1}{2\sqrt{1-\mu_0\mu_1}} \left(\frac{\partial U}{\partial\chi} = \frac{\partial U}{\partial\tau} = 0\right)$$

$$X = 0, \quad \vartheta = c\cos^2\chi_0, \quad \Omega = c\sin^2\chi_0, \quad R = \sqrt{2I^2(U+h)-c^2},$$

$$\vartheta = \vartheta_0 + c\int_{I_0}^{I} \frac{dI}{I\sqrt{2I^2(U+h)-c^2}}, \quad t = t_0 + \int_{I_0}^{I} \frac{IdI}{\sqrt{2I^2(U+h)-c^2}}$$

Dans ce cas:

$$r = r_0 = r_1 = \frac{\cos\chi_0}{\sqrt{\mu_2}} I = \frac{I}{\sqrt{\mu_2+\mu_3(1-\mu_0\mu_1)}}$$

$$U = F\left(\frac{\cos\chi_0}{\sqrt{\mu_2}} I\right) \sum m_i m_j$$

2. Cas colinéaire. Soit $F(r) = \pm r^{\alpha}$ $(\alpha \neq 0)$. Dans ce cas $I^{-\alpha}U$ est indépendante de I et l'on peut satisfaire aux équations (A) par la position $\tau = 0$, $\chi = \chi_0$, où χ_0 est une racine d'équation $[\partial(I^{-\alpha}U)/\partial\chi]_{\tau=0} = 0$ reliant les rapports des distances et des masses,

$$X = 0, \quad \vartheta = c\cos^2\chi_0, \quad \Omega = c\sin^2\chi_0, \quad R = \sqrt{2I^2(U+h)-c^2}$$

Dans ce cas

$$U = \pm \sum m_i m_j r_k^{\alpha} = AI^{\alpha}$$

A désignant une constante, et

$$\vartheta = \vartheta_0 + c\int_{I_0}^{I} \frac{dI}{I\sqrt{2AI^{\alpha+2}+2hI^2-c^2}}$$

$$t = t_0 + \int_{I_0}^{I} \frac{IdI}{\sqrt{2AI^{\alpha+2}+2hI^2-c^2}}$$

Dans le cas classique d'attraction suivant la loi de Newton $[F(r) = 1/r]$ les solutions 1 et 2 ont été obtenues par Lagrange (1). Laplace (2) a généralisé ces solutions pour les valeurs entières de α.

* Si $\alpha = 2$, l'intégration des équations (A) peut être effectuée à l'aide des fonctions élémentaires.

Dans le cas où $c=0$ (mouvement sur la droite immobile) on obtient une généralisation du cas d'Euler (3):

$$\tau=0,\quad \chi=\chi_0,\quad X=\vartheta=\Omega=0,\quad R=I\sqrt{2AI^{\alpha}+2h}$$

$$\vartheta=\vartheta_0,\quad t=t_0+\int_{I_0}^{I}\frac{dI}{\sqrt{2AI^{\alpha}+2h}}$$

3. Cas isoscèle. Soit $F(r)=\pm r^{-2}$. Les deux dernières équations du système (A) sont alors vérifiées par

$$\tau=-\frac{\pi}{2},\quad \vartheta=\Omega=0\quad\left(c=0,\ \vartheta=\vartheta_0,\ \omega=\vartheta_0+\frac{\pi}{2}\right),\quad m_0=m_1$$

on a donc

$$r_0=r_1\quad\text{et}\quad\frac{\partial U}{\partial\tau}=0$$

On obtient, en vertu des deux premières équations (A):

$$X\frac{dX}{\partial\chi}=\frac{\partial(I^2U)}{\partial\chi}$$

(I^2U ne dépend que de χ) et

$$X=\sqrt{2I^2U+c_1},\quad R=\sqrt{2hI^2-c_1}$$

où c_1 est une constante d'intégration.

D'après la première des équations (A)

$$I\frac{d\chi}{dI}=\frac{\sqrt{2I^2U+c_1}}{\sqrt{2hI^2-c_1}},\quad \int_{\chi_0}^{\chi}\frac{d\chi}{\sqrt{2I^2U+c_1}}=\int_{I_0}^{I}\frac{dI}{I\sqrt{2hI^2-c_1}}$$

et, enfin,

$$t=t_0+\int_{I_0}^{I}\frac{I\,dI}{\sqrt{2hI^2-c_1}}=t_0+\frac{1}{2h}\left(\sqrt{2hI^2-c_1}-\sqrt{2hI_0^2-c_1}\right)$$

Le problème se ramène aux fonctions elliptiques. Ce cas a été traité par Woronetz (4) et a été généralisé par l'auteur (5).

4. Cas de Jacobi. Dans le cas où $F(r)=\pm r^{-2}$ et $\tau=0$, $\vartheta=\Omega=0$ ($c=0$, $\vartheta=\omega=\vartheta_0$), on obtient pour X, R, χ, t les expressions analogues à celles du cas 3, avec une autre dépendance I^2U de χ.

Ce problème fut, pour la première fois, ramené aux quadratures par Jacobi (6).

5. En prenant $F(r)=\pm r^4$ [$f(r)=\pm 4r^3$, $\alpha=4$] et, en outre, $m_0=m_1=m_2=m\left(\mu_0=\mu_1=\frac{1}{2},\ \mu_2=\frac{m}{2},\ \mu_3=\frac{2}{3}m\right)$, nous obtiendrons un nouveau cas d'intégrabilité.

En posant, comme plus haut, $\tau=0$, $\vartheta=\Omega=0$ ($c=0$, $\vartheta=\omega=\vartheta_0$), considérons le mouvement rectiligne des trois corps.

Dans ce cas $U=\pm\frac{9}{2}I^4$ ($\partial U/\partial\chi=0$), et la seconde équation (A) donne $X=\text{const}=X_0$, de sorte que

$$R=\sqrt{\pm 9I^6+2hI^2-X_0^2}$$

et

$$d\chi=\frac{X_0\,dI}{I\sqrt{\pm 9I^6+2hI^2-X_0^2}},\quad dt=\frac{I\,dI}{\sqrt{\pm 9I^6+2hI^2-X_0^2}}$$

On en tire

$$I^2 = \wp(3t + a) = \wp(u)$$

où a est une constante d'intégration, et la fonction de Weierstrass a pour invariantes $g_2 = \mp \frac{8}{9} h$, $g_3 = \pm \frac{4}{9} X_0^2$ et

$$\chi = \frac{X_0}{3} \int \frac{du}{\wp(u)} + b$$

où b est une nouvelle constante d'intégration. Le problème est ainsi ramené aux fonctions elliptiques.

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LITTÉRATURE CITÉE

[1] J. Lagrange, Essai sur le problème des trois corps, Oeuvres, VI, 1772. [2] P. Laplace, Traité de mécanique céleste, X, ch. VI, 1805. [3] L. Euler, Novi Commentarii Ac. Sc. Imp. Petropol., XI (1765). [4] П. В. Воронец, Универс. известия, XVII, № 1—2, Киев (1907). [5] Ю. Соколов, Збірник праць Інст. мат. АН УРСР, № 3 (1939); № 5 (1940); № 6 (1941). [6] C. Jacobi, Problema trium corporum mutis attractionibus cubis distantiarum inverse proportionalibus recta linea se moventium, Gesamt. Werke, IV.

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AERODYNAMICS

DETERMINATION OF LIFT ON AEROFOIL UPON SEPARATION OF FLOW FROM ITS SURFACE

By O. V. GOLUBEVA

(Communicated by L. S. Leibenson, Member of the Academy, 12.V.1944)

One of the most important characteristics of an aerofoil is the maximum value of its lift, which determines the landing speed and the maneuverability of the airplane.

The present paper is a first attempt to secure a means of establishing this characteristics. It suggests an approximate method for the calculation of the lift on an aerofoil in the case of separation of flow from its surface.

1. In order to solve the problem in an approximate way it will suffice to consider a flow of an ideal fluid past an aerofoil in the presence of a stable vortex street directed along the flow.

The breadth of the vortex street h is supposed to be equal to the projection of the distance between the trailing edge and the separation point on to the direction perpendicular to the incoming flow. In computing the lift we will make use of Zhukovsky's theorem

$$P = \rho v_\infty \Gamma \tag{1}$$

which has been generalized by Taylor to comprise the case of separation [1].

2. Let us express the intensity $\Delta\Gamma$ of the vortex street through its breadth h. It follows from the conditions of stability that

$$\bar{u} = \Delta\Gamma/l\sqrt{8}, \quad h/l = 0.281 \tag{2}$$

where $\bar{u}$ is the velocity of the vortex street; l is distance between two neighbouring vortices.

If the quantity $\bar{u}$ be replaced by Strukhal's number

$$\mathrm{Sh} = nh/v_\infty \tag{3}$$

where $n = \frac{v_\infty - \bar{u}}{l}$ (n, the frequency with which vortices are shed; v_∞, the velocity of the incoming flow), we shall get

$$\Delta\Gamma = 10.1\ hv_\infty\ (1 - 3.55\ \mathrm{Sh}) \tag{4}$$

Formula (4) can be simplified. Experiments show that the number Sh is practically invariable with the breadth of the vortex street. Let us therefore consider a limiting case where $\Delta\Gamma$ is equal to the vortex Γ_0 forming at the start of motion

$$\Gamma_0 = 4\pi v_\infty R \sin(\alpha + \beta) \tag{5}$$

In the general case $\Delta\Gamma$ is not equal to Γ_0, yet taking Γ_0 to be always the intensity of the initial vortex at the trailing edge, we shall find for $\alpha + \beta = \pi/2$ that $\Delta\Gamma = \Gamma_0$ by virtue of symmetry.

The chord of the aerofoil for $R=1$ is approximately equal to 4. Then $Sh=0.196$, which agrees well with the experiment ([2]). And the approximate formula for $\Delta\Gamma$ is

$$\Delta\Gamma=\pi v_\infty h \tag{6}$$

3. The circulation around the aerofoil may be determined from the condition that the velocity at the trailing edge is finite. Theoretically the velocity at the sharp edge will be finite only at the instant when the vortex is shed off, or at the instant when the circulation of the vortex stops growing.

Let us consider the position of the vortices in the vortex street with respect to the aerofoil, in the case of finite velocity at the trailing edge. Closest to the aerofoil is the vortex formed from the trailing edge. The other vortices of the street are arranged according to Karman's condition of stability.

The chosen arrangement of the vortices will correspond to the optimal value of circulation.

From the condition that the velocity is finite at the trailing edge we get the following expression for the optimal circulation Γ_{opt}:

$$\Gamma_{opt}=\Gamma_0-k\Delta\Gamma \tag{7}$$

where

$$k=\sum_{n=0}^{\infty}\left[\frac{\rho_p^2-1}{\rho_p^2+1-2\rho_p\cos(\theta_p+\beta)}-\frac{\rho_p'^2-1}{\rho_p'^2+1-2\rho_p'\cos(\theta_p'+\beta)}\right]$$

ρ_p and θ_p are polar coordinates of the centres of the vortices in the lower row of the vortex street, on an auxiliary plane forming circumferences; ρ_p' and θ_p' are the respective coordinates for the upper row of the vortex street.

If the angles of incidence range within $15-20^\circ$ and the concavity from the chord of the aerofoil is 2—3 per cent, k can be expressed approximately by a simpler series:

$$k=\sum_{n=0}^{\infty}\left[\frac{\rho_p+1}{\rho_p-1}-\frac{\rho_p'+1}{\rho_p'-1}\right]$$

and besides we have

$$\rho_p=\rho_0+nl,\quad \rho_p'=\rho_0+\left(\frac{1}{2}+n\right)l \tag{8}$$

where ρ_0 is the distance to the nearest vortex from the centre of the auxiliary circumference.

The determination of ρ_p and ρ_p' from the formulae (8) gives a result that deviates less than 7 per cent from the true values of ρ_p and ρ_p'. The ultimate expression of k is in the form of a convergent series:

$$k=\frac{1}{l}\sum_{n=0}^{\infty}\frac{1}{n^2+\left(\frac{2}{l}-\frac{2\rho_0}{l}+\frac{1}{2}\right)n+\left(\frac{(\rho_0-1)^2}{l^2}+\frac{\rho_0}{2l}-\frac{1}{2l}\right)}$$

As Sh is a constant number, so the larger is h (or l), the less frequently do the vortices spring from the surface, and, accordingly, the centre of the vortex at the instant of separation is the farther away from the aerofoil.

Assuming a linear dependence between ρ_0 and l

$$\rho_0=a(\alpha)\,l+1$$

we find

$$k=\varphi(\alpha)/l \tag{9}$$

where $\varphi(\alpha)$ is a universal function for all aerofoils and varies only with the angle of incidence.

4. Let us consider the relation

$$\frac{\Gamma_0 - \Gamma_{mean}}{\Gamma_0 - \Gamma_{opt}} = f(l)$$

where Γ_{mean} is the mean value of the circulation around the aerofoil. This relation depends only upon l, since Γ_{mean} and Γ_{opt} do not differ from each other except in the position of the vortices with respect to the aerofoil, while the direction of the vortex street is invariable.

Supposing $f(l)$ to be a linear function of l, it may be shown that its free term is equal to nought, and $f(l) = Al$. Quantitatively the direct proportionality is borne out by experiment. The larger, in fact, the zone of separation, the wider the area over which are scattered the points obtained experimentally on measuring the lift. In other words, the deviation of the mean value of the lift from the optimal value increases with the zone of separation.

Ultimately we have

$$\Gamma_{mean} = \Gamma_0 - A\varphi(\alpha)\Delta\Gamma = \Gamma_0 - k_1\Delta\Gamma \qquad (10)$$

Let us examine the extremal values. The maximum value of Γ_{mean} corresponds to $\Delta\Gamma = 0$, the case where the flow past the aerofoil is without separation and the circulation is determined from Zhukovsky's hypothesis. Γ_{mean} is a minimum when $\Delta\Gamma$ is a maximum, that is for the largest region of separation, which always takes place with a particular kind of aerofoil, namely, an infinitely thin plate.

Let us assume that in the case of complete separation of streamlines from the aerofoil Γ_{mean} coincides with the Kirchhoff solution for the plate. As $\Delta\Gamma = \Gamma_0$ in this case, therefore we obtain for k_1 the formula

$$k_1 = 1 - \frac{\cos\alpha}{4 + \pi\sin\alpha} \qquad (11)$$

5. Let us determine h by successive approximations. From the pressure distribution corresponding to flow without separation at a given angle of incidence we find the separation point of the boundary layer in the upper surface. This point gives the first approximation of $h^{(1)}$. The corresponding $\Delta\Gamma^{(1)}$ and $\Gamma^{(1)}_{mean}$ may then be determined.

Further approximations will somewhat refine the value of $h^{(1)}$.

In order to obtain the distribution of pressure across the aerofoil for the second approximation, we can replace the entire vortex street by two vortices (the vortices located far away from the aerofoil have very slight effect on pressure distribution). In order to obtain an average for the distribution of pressure, we shall suppose the vortices of intensity $\Delta\Gamma^{(1)}$ to be located symmetrically at a distance $h^{(1)}$ with respect to the auxiliary circumference; and the circulation around the aerofoil we take to be equal to $\Gamma^{(1)}_{mean}$. The distance of the vortices from the centre of the circumference r is found from the condition that the velocity be finite at the trailing edge. We obtain the following equation from which r can be determined:

$$k_1 = \frac{r^2 - 1}{r^2 + 1 - 2r\cos(\theta_1 + \beta)} - \frac{r^2 - 1}{r^2 + 1 - 2r\cos(\theta_2 + \beta)}$$

where $\theta_1 = \alpha - \varphi$, $\theta_2 = \alpha + \varphi$, and $\sin\varphi = h/2r$.

Hence we can assume for angles of incidence less than 30° that

$$r = 2\cos(\alpha + \beta) + \sqrt{\frac{2h\sin\alpha}{k_1}} \qquad (12)$$

From the new pressure distribution we find the new separation point and the second approximation of $h^{(2)}$, and so on. This method of determining the lift has the important advantage of taking into account the individual features of a particular aerofoil.

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REFERENCES

[1] G. I. Taylor. ARC. R and M. 989 (February 1924). [2] Г. Н. Петров и Р. И. Штейнберг, ЦАГИ, вып. 482 (1940).

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PHYSICS

ON VELOCITY DISTRIBUTION IN DIRECTIONAL EXPLOSION

By G. I. POKROVSKY

(Communicated by P. L. Kapitza, Member of the Academy, 29.IV.1944)

In a directional explosion connected with the presence of a depression in the charge of the explosive (1) there is known to form a cumulative jet, the head part of which travels at a very high speed (2).

The speeds in the cumulative jet in the axial direction as will as perpendicular to the axis have been investigated by the author following the method developed by L. L. Dashkevich. The jet was photographed, through a narrow slit, on a rotating film disc. The slit had its images projected along the radius of the disc. From the distortion of the photographs caused by the rotation the speed of movement of the luminous parts of the jet could be measured. The method is particularly efficient when the explosion of charges enclosed in an aluminium shell is the object of study: the shivered metal of the shell burns out in the stream of the explosion products lending a remarkable brightness to the flame of explosion. In the tests whose results are reported below electric detonators No. 8 were used, and the charge, provided with a cumulative depression, was enclosed in an aluminium shell.

1. The maximum speed of the luminous part of the cumulative jet is 7660 ± 200 metres per second when the cumulative depression has a continuous shell.

2. If there is no shell about the depression, the maximum speed of the luminous portion is 12500 metres per second.

3. The maximum speed of that part of the jet which though dark is yet able to break through metal is 23000 ± 2000 metres per second, if there is a central hole in the shell about the cumulative depression. The diameter of the hole was about $^1/_8$ the diameter of the depression.

In this case the jet was recorded by the torch-like outburst of flame observed upon piercing an aluminium sheet 0.05 cm in thickness.

4. The speed of the lateral expansion of the jet (measured when the slit is at right angles to the jet axis) is 400 ± 100 metres per second.

5. The speed of the explosion products in the immediate vicinity of the lateral part of the charge is 3000 ± 200 metres per second.

6. The speed of the tail portion of the cumulative jet is quite insignificant (several tens of metres per second).

It appears from these data that the head portion of the cumulative jet moves at a very high speed, whereas the tail is all but stationary. The lateral expansion of the jet is rather slow also. Thus, in the head part of the jet we have very considerable quantities of forward motion energy concentrated in a small mass of explosion products.

The density of this energy (proportional to the square of the velocity) is several tens of times as great as that of the energy stored in the charge of the explosive.

Engineering Committee
of the Red Army.

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REFERENCES

(1) Г. И. Покровский, Направленное действие взрыва, 1942. (2) F. A. Korolev and G. I. Pokrovsky, C. R. Acad. Sci. URSS, XLII, No. 6 (1944); ЖЭТФ, № 8 (1944); G. I. Pokrovsky and K. P. Staniukovich, Bull. Acad. Sci. URSS, sér. phys., VIII, No. 4 (1944).

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GEOPHYSICS

EXPERIMENTS ON PROPAGATION OF SOUND IN TURBULENT ATMOSPHERE

By V. A. KRASSILNIKOV

(Communicated by O. J. Schmidt, Member of the Academy, 3.V.1944)

Some preliminary experimental observations by the interference method on the speed of sound in a free atmosphere (1) have led to results that are interesting both in point of method and as throwing some light on the character of the atmospheric turbulence. In further developing the research it was found necessary to leave the subjective method of observing a phasic picture on the screen of a cathode ray tube, and turn instead to recording the phase fluctuations in order to arrive at definite quantitative results.

After several circuits had been tried a phasemeter with a cathode ray tube was constructed. One arm of the phasemeter by means of a bridge composed of capacity and resistance sent a circular scanning to the cathode ray tube. The other arm cre-

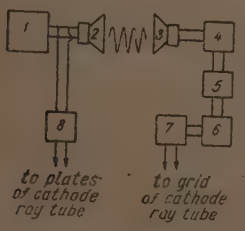

Fig. 1. Block-diagram No. 1

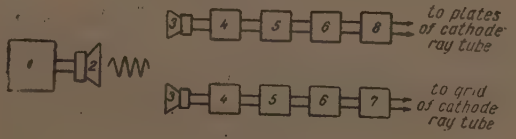

Fig. 2. Block-diagram No. 2

ated sharp positive voltage pulses—four in one period, which were transmitted to the grid of the cathode ray tube.

The cathode ray tube was non-conducting owing to a high negative voltage on its grid. It was made a conductor by positive pulses delivered four times in a period. In that way four luminous points instead of a circle appeared on the screen of the cathode ray tube, and they moved in a circle when the phase difference of the signals transmitted to the arms of the phasemeter was changed. Of the whole screen area of the tube only a 100° sector was left open, whose centre coincided with the centre of the circle (the trajectory of the four points). Thus, in this sector there were either one point or two and no more than two (at the edges of the sector). In front of the screen a cinematographic film was stretched which moving at a rate of 1 cm per second recorded the fluctuations of the phase. The motion picture photographs were taken with the FED objective 1 : 2. The method ensured a single-valued recording of the phase variation.

In the spring of 1941 a series of tests on the propagation of sound was made in the acoustic experiment field of the Palace of Soviets' Building Administration at Khlebnikovo near Moscow. There were obtained a number of records of the phase and amplitude of the sound as related to the sound frequency f, the length of the base between the microphones B and the average velocity of the wind $\bar{v}$. The scheme of the experiments is shown in Figs. 1 and 2.

These figures represent the block-diagrams adopted for measuring the variation of the phase between the loud-speaker and the microphone (Fig. 1) and between two microphones (Fig. 2), respectively. The arabic numerals in the figures mean: *1*, sound generator; *2*, loud-speaker; *3*, microphone; *4*, octave filtre; *5*, amplifier; *6*, voltage stabilizer; *7*, pulse arm of the phasemeter; *8*, circular scanning arm of phasemeter. With scheme No. 1 distance from microphone to loud-speaker was 140 m ($ML = 140$ m); the loud-speaker was installed on a 25 m tower, and the microphone 10 m high above the earth (loose snow). In the case of scheme № 2 the distance from the loud-speaker to the middle of the base, ML, was 20 m. Both the microphones and loud-speakers were set on stands at a height of 8 m.

The intensity of the sound fluctuated so greatly, especially in a strong wind and at high frequencies ([2],[3]), that it was found necessary for the correct work of the phasemeter to place voltage stabilizer (of the ordinary type with varima tubes) on either of its arms.

The oscillations of the sound amplitude were recorded on the film by means of short-period galvanometers (KPG-2).

As a result of a statistical treatment of the obtained data the following conclusions have been arrived at.

1. The fluctuations of the phase at frequencies ranging within 3000 to 4000 cycles are very considerable, even if the speed of the wind is not very great [the mean square of the phase difference $\sigma\varphi$ at 1 to 8 m long base (block-diagram No. 2) varies from 24 to 128° for $ML = 20$ m; $f = 4000$ cycles, $\bar{v} = 2.7$ m/sec.] and obey a distribution law that is close to that of Gauss' distribution.

2. The relative variation of the amplitude under the above conditions may amount to 50 per cent of the mean value. The coefficient of correlation between the two values of the amplitude at the ends of the base B declines with the base, and for $B = 8$ m the variations of the amplitude may be looked upon as independent of one another.

3. The mean square of the phase fluctuation $\sigma\varphi$ is proportional to the mean velocity of the wind $\bar{v}$.

4. It is also proportional to the frequency of the sound in the range 2000—5000 cycles.

5. The mean square of the phase difference at the ends of the base B grows with the base as $B^{5/6}$.

6. The relative variation of the phase $\sigma\Delta_t\varphi/\sigma\varphi$ during the intervals 0.1, 0.2, . . . , 1 sec. fluctuates from 0.91 to 1.4 for $B = 1$ m ($f = 4000$ cycles, $\bar{v} = 2.7$ m/sec.); from 0.48 to 1.4 for $B = 4$ m.

7. In the case of block-diagram No.1 $\sigma\Delta_t\varphi = 18°$ for 0.1 sec. when $f = 800$ cycles, $ML = 140$ m, $\bar{v} = 6.5$ m/sec., $\bar{v} \perp ML$.

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REFERENCES

[1] V. M. Bovsheverov, V. A. Krassilnikov, C. R. Acad. Sci. URSS. XXXII, No. 7 (1941). [2] H. Dahl, O. Devic, Nature. 139, 550 (1937). [3] H. Sieg, Elektrische Nachrichten Technik, 17, 9 (1940).

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CHEMISTRY

COMPLEX COMPOUNDS OF PLATINUM WITH ACETYLENE DERIVATIVES

By A. GELMAN, S. BUKHOVETZ and E. MEILAKH

(Communicated by I. I. Chernjaiev, Member of the Academy, 15.VI.1944)

In an investigation of complex compounds of platinum with ethylene and its homologues A. Gelman has found out that ethylene hydrocarbons exert in the inner sphere a high trans-influence (1). The further study has shown that similar behaviour is exerted by carbon monoxide (2) and nitrous oxide (3).

It could be supposed that acetylene and its derivatives, being unsaturated molecules, would also exert a high trans-influence in the inner sphere. However, reiterated attempts by A. Gelman and by Chernjaiev and Litvak to prepare definite complex compounds of platinum with acetylene ended in failure. Brownish substances obtained in the reaction of potassium chloroplatinite solutions with acetylene were of quite an indefinite composition, and practically insoluble in water as well as in organic solvents.

On the supposition that acetylene, owing to its acid properties, could yield acetylides with platinum salts, it was decided to investigate such acetylene derivatives, in the case of which the formation of acetylides would be out of the question. For this purpose 2,5-dimethyl-3-hexyne-2,5-diol was selected. This acetylenic γ-glycol is well soluble in water and comparatively easily accessible.

Preparation of trans-$[PtC_8H_{14}O_2PyCl_2]$. When a concentrated aqueous solution of potassium chloroplatinite was heated with 2,5-dimethyl-3-hexyne-2,5-diol for 15—20 min., the solution became yellow, and there separated a small amount of a brown precipitate. The cooled solution was filtered 2—3 times through the same filter until it became quite transparent.

The reaction was believed to have run according to the equation:

$$K_2PtCl_4 + \begin{matrix}CH_3\\OH\\CH_3\end{matrix}\!\!>\!C-C\equiv C-C\!<\!\begin{matrix}CH_3\\OH\\CH_3\end{matrix} = K[PtC_8H_{14}O_2Cl_3] + KCl$$

Pyridine was added carefully (drop by drop) to the yellow-red filtrate, a yellow precipitate appearing immediately. In the majority of cases this precipitate was a sticky and oily substance which, when cooled with snow or water and triturated with a glass rod, crystallized rapidly. The precipitate was recrystallized from benzene or ether in the cold. Analysis showed that the reaction had proceeded according to the following equation:

$$K[PtUnCl_3] + Py = [PtUnPyCl_2] + KCl$$

(the substance analysed had been dried in a dessicator to constant weight, see Table 1).

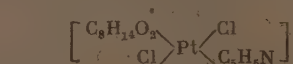
Table 1

$[PtC_8H_{14}O_2C_5H_5NCl_2]$, mol. weight 487.25

Pt, %		Cl, %		N, %		C, %		H, %		O, %	
Calc.	Found	Calc.	Found	Calc.	Found	Calc.	Found	Calc.	Found	Calc.	Found
40.07	40.10 40.04	14.56	15,12	2.87	2.42 2.60	32.02	32.31 32.50	3.92	4.00 4.17	6.56	6.03

The properties of the compound were investigated in detail and are reported in Table 2.

Table 2



Outward appearance	Fine yellow crystals of indefinite form
Indices of refraction*	$N_g = 1.682$ $N_p = 1.591$
Solubility in water at 25° C	7.4 g per 100 g of solution
Solubility in benzene at 25° C	9.1 g per 100 g of solution
Electroconductivity of aqueous solution at 25° C**	ρ, l/mole: 500 1000 2000 4000 8000 μ 16.85 18.66 21.06 26.13 29.53
Melting point in an open tube	96—97° C
» » in a sealed tube	80—81° C
Relation towards dilute CHl in the cold	Dissolves slowly with the formation of PyH $[PtUnCl_3]$
Relation towards thiourea	Dissolves rapidly with the formation of $Pt4Sc(NH_2)_2Cl_2$
Behaviour towards hot water	Decomposes with the separation of a brown precipitate
Molecular weight determined by the cryoscopical method, in benzene	497.1 495.44 calculated 487.25; error 1.8%

Determined by E. E. Burovaya. ** Determined by V. A. Ezuchevskaya.

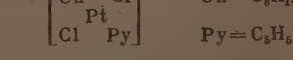
As may be seen from Table 2, the chemical behaviour of a platinum complex that has a substituting group with an acetylenic bond in its molecule is quite similar to that of ethylene-dichloro-diamine platinum possessing trans-configuration.

Conclusions. 1. The possibility has been demonstrated of preparing a complex compound of platinum with an acetylenic γ-glycol having the composition $[PtC_8H_{14}O_2PyCl_2]$.

2. The study of the properties of this dichloride has shown that the acetylenic bond as well as the ethylenic one exerts an enhanced trans-influence in the inner sphere of the complex.

3. The dichloride is a monomer, this has been proved by the determination of its molecular weight.

4. From its chemical behaviour the dichloride appears to have a trans-configuration:



$Un = C_8H_{14}O_2$

$Py = C_5H_5N$

The investigation is being continued.

Institute of General and Inorganic Chemistry,
Academy of Sciences of the USSR.

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REFERENCES

[1] А. Гельман, Известия сект. по изуч. платины, вып. 16 (1939). [2] A. D. Gelman and M. M. Bauman, C. R. Acad. Sci. URSS, XVIII, No. 9 (1938). [3] A. D. Gelman and Z. P. Maximova, ibid., XXIV, No. 8 (1939).

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PHYSICAL CHEMISTRY

ON THE EXPLOSIVE PROPERTIES OF «GASLESS» SYSTEMS

By A. F. BELAJEV and A. B. NALBANDJAN

(Communicated by N. N. Semenov, Member of the Academy, 18.III.1944)

The explosive properties of any substance are usually characterized by the following three basic factors taken together:

1) rapid transformation of the substance into the end products of the reaction;
2) presence in the transformation products of gases or vaporous substances;
3) the exothermal character of the transformation.

These three factors taken together render the system explosive. The exothermal character of the explosive transformation insures the high temperature of the gas or vapour produced as a result of the explosive reaction. The gases or vaporous substances that form as a result of the explosive transformation are the working substance which transforms the thermal energy of decomposition into mechanical work. The instantaneous nature of the transformation of an explosive substance into the end products insures the high pressure of the explosion gases that perform the work of destruction.

Quite essential for the theory of explosives is a knowledge of the rôle of each of the three above-mentioned factors separately.

Whereas the rôle of the instantaneous nature of the transformation and the heat of reaction is obvious, it is not clear what effect the volume of the gaseous products has. According to Berthelot, the explosive properties of a substance, its capacity to perform work, are determined by the so-called «characteristic» product QV_0, and hence the heat of reaction and the volume of the gaseous products are of equal importance. Explosive action, according to Berthelot, remains constant if, for example, while decreasing the heat of reaction by half one correspondingly increases the volume of the gaseous products, and *vice versa*. This conclusion is in contradiction to experiment, and Berthelot's viewpoint may be regarded as having only an historic interest.

Schmidt defines the maximum capacity for doing work as $A = Q - q$, *i. e.* as the difference between the total heat of explosion and the heat remaining in the reaction products, when the pressure reaches atmospheric and expansion ceases. In such an appraisal of the maximum capacity for doing work, the rôle of the quantity V_0—the volume of the gases—should be very insignificant. Schmidt points out, for example, that for definite conditions a two-fold increase in the quantity of the gases increases the capacity for doing work only by 6 per cent. To obtain the most sharply defined effect of the volume of the gases on the rate of detonation, direct action and capacity to perform work, we set up a number of experiments with explosive mixtures that would not yield any gaseous products in the final stage.

Such systems are, for example:

$$3.6\ KClO_3 + FeSi_{4.65} = 3.6\ KCl + 0.5\ Fe_2O_3 + 4.65\ SiO_2$$

$$KClO_3 + 2Al = KCl + Al_2O_3$$

which, as is seen, do not contain gaseous products either in the initial or in the final stage. Hence they were chosen for our investigation. It should be pointed

out that as regards the mixture $KClO_3 + Al$, informations on its ability to explode are available in the literature.

After drying the potassium chlorate was passed through a No. 30 sieve (30 meshes per linear cm), and then stoicheiometric mixtures with finely pulverized ferrosilicon and aluminium, respectively, prepared. The mixtures were mixed in small (100 g) portions in a small porcelain ball mill ($V = 2$ l) with porcelain balls for 1.5 hrs, after which they were regarded as ready for an experiment.

1. Experiments with the Mixture $KClO_3 + FeSi$. Theoretically stoicheiometric mixture of $KClO_3 + FeSi$ can liberate upon explosion about 1750 cal/kg.

In Table 1 there are given compressions of a lead column obtained by employing a modified Hess method with the usual lead column and usual steel plates, but different diameters of the charge. The charge density was equal to $\rho = 0.94$ g/cm^3.

Detonation of the mixture was initiated by a No. 8 capsule-detonator.

Table 1

Diameter of charge	Weight of charge, gs	Compression of lead column, mm
40	50	0
58	240	0.7
77.5	480	15.1

Table 2

Diameter of charge	Weight of charge, gs	Compression of lead column, mm
40	50	5.40
58	240	16.73
77.5	480	21.55

It is seen that in a cartridge 77.5 mm in diameter it is possible to initiate detonation in the «gasless» mixture $KClO_3 + FeSi$ and to obtain a considerable compression of the column.

Further experiments on determining the stability of the detonation and its velocity were carried out in metallic tubes of different diameters. The experiments showed that in tubes with a diameter greater than 16 mm the detonation induced by the capsule-detonator is quite stable, and proceeds with a certain acceleration along the entire length of the tube (40 cm). In tubes with a diameter less than 16 cm the detonation dies out before reaching the end of the tube. The detonation velocity measured by the Dotrisch method in a tube 20 mm in diameter was found to be equal to 1170 m/sec. In a tube 27 mm in diameter $v = 1270$ m/sec.

2. Experiments with the Mixture $KClO_3 + 2Al$. The heat of explosion of 1 kg of this mixture is approximately equal to 2200 cal. In Table 2 there are given the compression of a lead column in mm as a function of the charge, and its diameter for a mixture density of $\rho = 0.95$ g/cm^3.

As follows from the table, in this case a charge 50 g in weight already detonates. Compression, however, is very small here. In order to elucidate whether this small value of the compression is connected with the extinction of detonation, we took a charge of the same diameter but 150 g in weight, *i. e.* three times as long. The compression of the lead column was found to be 8.56 mm. This points to the fact that in a charge 40 mm in diameter detonation is quite stable, and the small value of the compression is due to the small value of the local action of the mixture.

Most characteristic are the results of measuring the local action of a «gasless» mixture of $KClO_3 + 2Al$ 50 g in weight placed in an iron ring 40 mm in diameter, as compared with the local action of the same quantity of a stoicheiometric mixture $KClO_3 +$ peat, which after explosion produces approximately 400 l of gases per 1 kg of the mixture. For the $KClO_3 + 2Al$ mixture we get a value of 18.40 mm for the compression of the lead column, and for the $KClO_3 +$ peat mixture, 18.0 mm. Just as in the experiments with the $FeSi + KClO_3$ mixture, the stability of the detonation and its velocity in metallic tubes of different cross-sections were determined.

In tubes with a diameter greater than 16 mm the detonation is stable along the entire length of the tube. In tubes having a diameter beginning with 16 mm and less the detonation rapidly dies out before reaching the end of the tube.

The velocity of detonation for a tube 21 mm in diameter was found to be equal to 1300 m/sec. In a tube 40 mm in diameter the corresponding value was 1370 m/sec.

3. Analysis of the Gaseous Explosion Products. The experiments to determine the volume of the gaseous explosion products for the system $KClO_3 + 2Al$ were carried out in a metal bomb. Into it was placed a charge 39.5 g in weight poured into an iron tube 40 mm in diameter, the bottom of which was covered with a lead cap. On the cover of the bomb was a device for exploding the mixture electrically and also a valve connecting the bomb with an oil pump. In order to do away with the interaction of foreign gases as well as the carbon that is formed during the explosion of the capsule-detonator, the latter was replaced by a charge of pressed lead azide 2.5 g in weight which was set off by a heated wire. Before exploding the mixture under investigation, the bomb was evacuated for 15—20 min. to 1 mm Hg pressure. The gas volume less the volume of the nitrogen formed during the explosion of lead azide was found to be 20 liters at atmospheric pressure per 1 kg of the $KClO_3 + 2Al$ mixture. Of this quantity about 45 per cent consisted of carbon-oxygen compounds (11 per cent CO and 34 per cent CO_2). The presence of carbon compounds in the gaseous explosion products is evidently due to the presence of carbon in the aluminium powder, the bomb walls, *etc.*

It is difficult, however, to conceive that a system yielding such an insignificant quantity of gases may produce a stable detonation and give the explosion effects considered above. It is necessary to take into account that a considerable part of the explosion work is expended at a temperature much higher than the critical temperature of KCl.

Hence it follows that KCl should be in the vapour state during a considerable portion of the explosion process, and consequently behave like a gas. (A similar conclusion on the rôle of vaporous KCl in the explosive action of the $KClO_3 + 2Al$ mixture is reached by N. N. Sokolov in his book on the theory of explosives). It is easy to calculate that approximately 125 liters of gases are formed per 1 kg of the $KClO_3 + 2Al$ and $KClO_3 + FeSi$ mixtures. Thus, the total quantity of gas formed in the case of the $KClO_3 + 2Al$ mixture is about 150 l/kg. This is already a significant quantity, but still much less than that produced by usual explosives. It is more correct, therefore, to term the systems considered here as systems with a small quantity of gases rather than «gasless».

In this connexion it may be noted that all our attempts to induce a stable detonation in actually gasless systems, which even at high temperatures do not yield any gases or vapours, were not successful, just as similar attempts of other investigators.

We performed probe experiments set up to compare the crater producing action of $KClO_3 + 2Al$ charges placed into the ground with the action of common explosives (amatol, dynamon-T). From these experiments the following preliminary conclusions may be drawn.

The relative crater producing action of the $KClO_3 + 2Al$ mixture is doubtless more powerful than would follow from the value of QV_0, which is small owing to the small value of V_0 (150 l/kg). On the other hand, this value does not correspond to the high energy value possessed by the system examined and is much smaller, which is naturally explained by the small quantity of gases. This large decrease in the crater producing effect, connected, in our opinion, with the influence of the quantity of gases liberated, is more considerable than follows from Schmidt's works. However, a complete and exhaustive treatment of the problem of the effect of the quantity of gases on the explosion work can be made only after more systematic and thorough experiments have been carried out.

In conclusion still another circumstance should be noted, namely, the relatively big acoustic report produced accompanying the explosion of the $KClO_3 + 2Al$ mixture.

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PHYSICAL CHEMISTRY

ON THE THEORY OF THE SPECIFIC HEAT OF HIGH POLYMERS

II. RUBBER, GRAPHITE

By V. V. TARASSOV

(Communicated by A. N. Frumkin, Member of the Academy, 15.X.1944)

As I have already shown elsewhere (1), the limiting laws of the variation of specific heat at low (Debye) temperatures for linear, two-dimensional and three-dimensional polymerization, differ in character.

The general expression I obtained for the specific heat at low temperatures (*i. e.*, under the condition that $T/\theta \ll 1$, where θ is the characteristic temperature) for a three-dimensional structure, gives the well-known Debye T^3-law.

$$c_3 = \frac{12}{5}\pi^4 R (T/\theta)^3$$

The limiting laws for two-dimensional nets of main valences and for chains of main valences are as follows:

$$\text{a)} \quad c_2 = 43.272 \cdot R (T/\theta)^2 \quad \text{for nets} \qquad (1)$$

$$\text{b)} \quad c_1 = \pi^2 R (T/\theta)^1 \quad \text{for chains} \qquad (1a)$$

In accordance with formula (1a) the linear change in the specific heat capacity of a linear high polymer is performed with an accuracy of up to 1 per cent when $T \leqslant \theta/8$, and of up to 10 per cent when $T \leqslant \theta/5.5$.

The case of two-dimensional, unlimited polymerization is described by the law of variation in specific heat given by formula (1). An attempt may be made to test the applicability of formula (1) by studying the low-temperature course of the specific heat of laminar lattices, for example. CdI_2, $CdCl_2$, etc. and also of graphite.

The Specific Heat of Graphite. We do not know of any experimental data on the low-temperature specific heat of laminar lattices. Such data are available only for graphite and this too, by the way, is far from being complete.

A curve of specific heat for Acheson graphite (graphite «lubricant») between 20 and 90°K, obtained by Simon and Swein (2), is given in the literature. We reproduce this curve here in Fig. 1.

Graphical interpolation of the specific heat along this curve shows that it obeys the T^2-law, and is in no agreement whatsoever with the T^3-law. This is clearly seen from the table. In the second column are given values of the atomic specific heat of graphite obtained by interpolating along the Simon curve. Highly convincing proof of the applicability of formula (1) is a comparison of the characteristic temperatures computed on the basis of this formula and the Debye T^3-law.

Heat Capacity and Characteristic Temperature of Graphite

m	According to Simon	Θ_2 (according to the T^2-law)	Θ_3 (according to the T^3-law)
30	0.03_8	1430	680
40	0.08_0	1310	720
50	0.12_7	1300	770
60	0.17_5	1330	830
70	0.22_5	1370	890
80	0.27_5	1410	950
90	0.33_8	1430	1000

The Specific Heat of Rubber. The biggest prospects for the proposed theory lie in its application to high-molecular chains of organic substances, in particular and perhaps especially, to rubber.

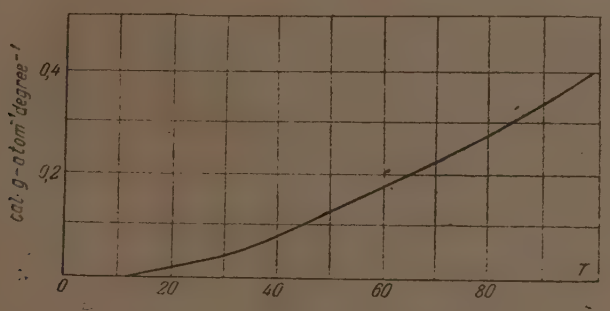

Fig. 1. The specific heat of graphite

Taking as a basis the precise measurements of Bekkedahl and Matheson [4], and Bekkedahl and Scott [5], who measured in an adiabatic calorimeter the specific heat up to 14° K of pure, natural rubber and synthetic rubber «Hycar-OR», we may attempt to determine whether or not the linear law in the (1a) form is applicable to the results they obtained. This, however, is fraught with a number of considerable difficulties. Here we point out only two of them: 1) The incompleteness of the experimental data does not permit us to recompute the experimental c_v to the necessary c_v. 2) The difficulties introduced by the fact that the chains of real high polymers consist of different atoms (polyatomic chains).

No single atomic chains are known to exist.

Development of the theory of the specific heat of crystals began with research into the applicability of the Debye function to single-atomic lattices. Only later a study was made of the possibility of interpreting the temperature course of the specific heat of polyatomic lattices.

In the same way it seems possible to separate in the total vibrational specific heat of a polyatomic, high molecular chain the part that is conditioned by the vibrations of the linear continuum and is described by formula (1a) [1]; from that due to the characteristic frequencies of the atoms or groups attached to the high molecular chain.

Below we give only a first attempt of this kind.

In Fig. 2 is given the temperature dependence of the specific heat of natural rubber. The detailed tabulated data from the above-mentioned works ([4], [5]) are not cited in the present paper.

The jump in the specific heat around 200° K for natural rubber and around 250° for Hycar-OR is associated with transformation in the neighbourhood of T_g.

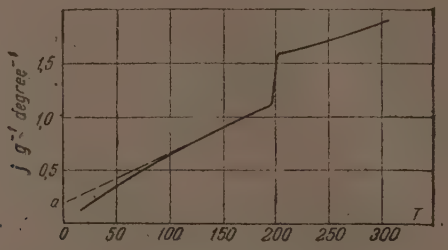

Fig. 2. The specific heat of natural rubber

Let us consider the premises we introduced in trying to interpret these experimental data:

1) The chain of carbon atoms linked directly by main valence forces in natural rubber obeys the «catena-function» c_1. In other words, it performs $3N$ characteristic (valence and deformational) vibrations following a linear distribution law([1]):

$$dS_1 = 3N\nu_{max}^{-1}\, d\nu \qquad (2)$$

2) The hydrogen atoms are attached to the chain of C atoms. The vibrations of the C—H bonds have a high frequency (2900—3000 cm^{-1}). They are very weakly excited and do not play an essential rôle in the specific heat balance.

3) Besides the vibrations of the chain of C atoms in natural rubber, rotational vibrations of the CH_2 and CH_3 groups occur. To those vibrations correspond rather low characteristic frequencies of the groups appended to the chain.

4) Owing to the diffuseness of the dynamic picture of the vibrations of the C-atomic chain, the treatment is conducted along two lines:

a) By assuming that in the polyprene chain there are for each isoprene residue four carbon atoms whose characteristic vibrations are distributed in the high molecular chain in accordance with the linear law (2). The fifth C atom in the «side chain» (CH_3 group) has his own characteristic frequencies of the valence and deformation vibrations of the C—C bond.

b) By assuming that in the polyprene chain all five C atoms in one isoprene residue effect their fibrations in the chain, obeying the linear law (2).

5) We assume that the correction for c_v up to the temperature of transformation is liable to no essential variation, and that therefore the slope of the linear segment of the specific heat curve up to 200° K coincides approximately with the slope of the curve describing the course of c_1.

6) For a high degree of polymerization of natural rubber the macromolecular factor ([3]) $3R/z$ may be put at zero.

At low temperatures (up to 100—120°) the curve is convex. This is explained by the occurrence in the given region of progressive excitation of the rotational (*i. e.* relatively low-frequency) vibrations of the chain and the CH_3 groups appended to it.

The transition to linearity signifies that the heat capacity of the indicated rotational vibrations has already been excited well to the limit.

From the slope of the linear segment of the specific heat curve for natural rubber we determine the characteristic temperature according to formula (1a).

For this we substract from the value (4) $c_{150^\circ} = 0.892$ joule·g^{-1}·degree^{-1} 0.17 corresponding to the segment $0-a$ in Fig. 2, and regard the difference obtained 0.722 joule·g^{-1}·degree^{-1} = 11.78 cal·mol.$^{-1}$ degree^{-1} as the vibrational «catena—specific heat»(1) of the carbon chain*. Following assumption 4a, *i. e.* assuming that only 4 C atoms effect vibrations according to the linear law (2), we get:

$$c^{(4)}_{150^\circ} = 11.78 : 4 = 2.95 \text{ cal/g. atom of carbon·degree}$$

Upon calculation according to (4b), we get;

$$c^{(5)}_{150^\circ} = 11.78 : 5 = 2.36 \text{ cal/g. atom of carbon·degree}$$

To these values $c^{(4)}_{150^\circ}$ and $c^{(5)}_{150^\circ}$, according to formula (1a), correspond the characteristic temperatures:

$$\left.\begin{aligned} \theta^{(4)}_{\text{cat}} &= \frac{\pi^2 R\, 150}{2.95} \approx 1000^\circ \\ \theta^{(5)}_{\text{cat}} &= \frac{\pi^2 R\, 150}{2.36} \approx 1250^\circ \end{aligned}\right\} \qquad (3)$$

It is most fortunate that we are in the possession of all the necessary data for computing the correction $c_p - c_v$ for the temperature $T = 300^\circ$:

$\alpha_{20-80^\circ} = 6.7 \cdot 10^{-4}$ (5); $\varkappa_{20-80^\circ} = 53.7 \cdot 10^{-6}$ cm^2/kg (7); $V_{300^\circ} = 1.104$ cm^3/g (6)

According to the well-known formula of the I and II laws we find:

$$c_p - c_v = \frac{\alpha^2}{\varkappa} TV = 0.275 \text{ joule·g}^{-1}\text{·degree}^{-1}$$

According to Bekkedahl (4) $c_{p\,300^\circ} = 1.890$ joule·g^{-1}·degree^{-1} whence:

$$c_{v\,300^\circ} = 1.615 \text{ joule·g}^{-1}\text{·degree}^{-1} = 26.28 \text{ cal·mol·degree}$$

Assuming that four of the rotational vibrations of the groups corresponding to four simple C—C bonds, on the isoprene residue at $T = 300$ are excited to the limiting value of the specific heat, to find $c_{\text{cat}\,300^\circ}$ we must subtract from the value $c_{v\,300^\circ}$ just obtained a quantity equal to $4R = 7.92$ cal/mol·degree. Thus

$$c_{\text{cat}\,300^\circ} = 18.36 \text{ cal/mol·degree}$$

In accordance with the two examined assumptions (4a) and (4b) the latter value $c_{\text{cat}\,300^\circ}$ gives:

$$\left.\begin{aligned} c^{(4)}_{\text{cat}\,300^\circ} &= 4.59 \\ c^{(5)}_{\text{cat}\,300^\circ} &= 3.67 \end{aligned}\right\} \quad \frac{\text{cal.}}{\text{g. atom of carbon, degree}}$$

From the graph and the table of catena-functions (1) corresponding to the values $c^{(4)}$ and $c^{(5)}$ we get the values of the characteristic temperature:

$$\theta^{(4)}_{\text{cat}} = 1010^\circ, \quad \theta^{(5)}_{\text{cat}} = 1500^\circ$$

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* The quantity $(0-a)$ may be taken to stand for $c_p - c_v$ and the specific heat of the rotational vibrations. Inasmuch as the latter are excited to a considerable degree and $c_p - c_v$ up to 200°K changes inessentially, the given simplified separation of the catena part of the vibrational specific heat of the C chain is admissible.

Interesting is the complete agreement of the value of $\theta^{(4)}$ obtained above (see formula 3) on the basis of the low-temperature law of the heat capacity with that obtained here, *i. e.* in the region of medium temperatures. This agreement speaks in favour of assumption 4a.

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REFERENCES

[1] V. V. Tarassov, C. R. Acad. Sci. URSS, XLVI, No. 1 (1945). [2] F. Simon u. R. C. Swein, Z. phys. Chem. (B), **28**, 189 (1935). [3] Landolt-Börnstein, Phys.-Chem. Tab., 2 Erg. Bd. 2 Teil, 5 Aufl., S. 1237, Berlin (1931). [4] N. Bekkedahl and H. Matheson, J. Res. Nat. Bur. Standards, **15**, 503 (1934); Rubber Chem. Techn., **9**, 264 (1936). [5] N. Bekkedahl and R. B. Scott, J. Res. Nat. Bur. Standards, **29**, 87 (1942); Rubber Chem. Tech., **16**, 310 (1943). [6] N. Bekkedahl, J. Res. Nat. Bur. Standards, **13**, 411 (1934); Rubber Chem. Tech., 8, 5 (1935); McPherson, J. Res. Nat. Bur. Standards, **8**, 751 (1932). [7] B. Scott, ibid., **14**, 99 (1935); Rubber Chem. Tech., **8**, 401 (1935).

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AGROCHEMISTRY

EFFECT OF SELENIOUS AND SELENIC ACIDS ON DEVELOPMENT OF PLANTS

By E. V. BOBKO and N. P. SHENURENKOVA

(Communicated by D. N. Prianishnikov, Member of the Academy, 19.VI.1944)

Our former investigations have shown that selenium when applied in sand cultures acts as a very strong poison on such plants as millet, mustard, horse-beans and the like. The poisonous effect is by far not so strong in soil cultures, apparently because the soil absorbs considerable quantities of selenium.

This year we have set ourselves the task of studying the development of plants when it is influenced by different degrees of oxidation of selenium. Generally, the poisonous effect of a chemical element decreases in the inverse order of its oxidation. Thus, the salts of sulphurous acid are more poisonous than those of sulphuric acid, and nitrous acid salts are stronger poisons than nitrates. The question arises, How the salts of selenium, which is an analogon of sulphur, will behave in this respect. For solving this problem we started experiments with millet and alfalfa in pots filled with sand or soil. Let us first discuss the results obtained with millet grown on sand. They are set out in Table 1.

Table 1

Different Degrees of Oxidation of Selenium as Affecting Millet Grown in Sand Culture Solutions (yield per pot in grams)

Se, mg/kg of sand	H_2SeO_3		Na_2SeO_4	
	Total yield	Grain	Total yield	Grain
0 (Hellriegel solution) . . .	38.36±1.11	11.31±0.19	38.36±1.11	11.31±0.19
0.5	41.73±0.51	14.16±0.04	42.56±0.01	13.05±0.95
1.0	39.06±1.49	13.25±0.05	54.83±1.26	18.16±1.84
2.0	37.21±0.99	13.60±0.42	53.66±0.41	19.00*
4.0	28.12±0.9	11.16±0.96	—	—

* Data on the yield of grain are available for one pot only.

In these and all subsequent experiments selenious acid was introduced in a free state, and selenic acid was introduced as Na_2SeO_4. The Hellriegel solution was added with a set of trace elements according to Hogland (mixture A—Z). Each pot had 5 kg of sand. The experiment was run in duplicate.

As may be seen from Table 1, both selenious and selenic compounds stimulated the development of millet. Plants with slight doses of selenium gave a higher yield than the control. Selenic acid (as sodium salt) was particularly efficient. In the case of selenious acid the first dose produced the best result. The higher doses had a less favourable effect, and with 4 mgs of Se per kg of sand the plants even gave a lower yield than the control.

The plants grown in the soil were by far less responsive to selenium. This is probably because a considerable portion of selenium is absorbed by the soil, and thereby converted in a form unavailable to the plant.

The mean response of plants to selenium in this case is evidenced by the results of the experiment with millet grown on moderately podsolized loam from the neighbourhoods of Moscow (Table 2). The pots were filled with 5 kg of absolutely dry soil each. The nutrient salts were supplied in the following quantities per pot: N, 0.5 g in the form of NH_4NO_3; P_2O_5, 0.3 g as Na_2HPO_4; K_2O, 0.4 g as K_2SO_4. Selenium was given in the form of Na_2SeO_4.

Table 2

Effect of Selenium on Soil Grown Millet

(yield per pot in grams)

Se, mg/kg of soil	Total yield	Grain
0 (NPK)	69.77±5.77	27.92±0.17
6.0	69.01±1.51	27.25±0.9
12.0	72.17±1.82	26.65±1.6
24.0	69.54±0.02	27.13±1.08
36.0	71.65±0.54	28.25±0.7
48.0	64.27±1.10	26.84±0.09

Table 2 shows that the dose of 36 mg of selenium had no effect on the development of the plants. Only with 48 mg of selenium did the plants give a certain increase of green mass. The variations in the yield of grain lie within the experimental error. This agrees with our former observations when it was found that vegetative rather than reproductive development was the first to suffer from the injurious action of selenium.

The next experiment was run with alfalfa in pots with soil. The plants were cut at the end of full blossoming. The size of the pots, the soil and the fertilizers were the same as in the millet experiment. The results are given in Table 3.

Table 3

Effect of Different Degrees of Oxidation of Selenium on Soil Cultures of Alfalfa

(yield per pot in grams)

H_2SeO_3		Na_2SeO_4		
Se, mg/kg of soil	Total yield	Se, mg/kg of soil	Total yield	Content of Se in plants, mg/kg
0 (NPK)	22.77±1.01	0 (NPK)	21.33±1.03	
2.0	28.38±0.68	6.0	20.31*	
4.0	26.02*	12.0	20.05±1.27	4.9±0.6
16.0	26.12±0.1	24.0	21.73±0.95	7.4±0.4
24.0	21.06±1.06	36.0	21.60±0.20	
		48.0	21.79±1.01	

* No replicates.

As is seen from Table 3, doses of up to 16 mg of Se had pronouncedly stimulating effect on the plants. With 24 mg of selenium the yield was somewhat below that of the control. But the difference lies within the error of the experiment, and we can only surmise that if a dose as high as 48 mg had been tried, the negative effect of selenious acid might possibly have become apparent. When sodium selenate was used as the source of selenium, the development of plants was neither depressed nor stimulated. Estimation of selenium in alfalfa plants supplied with 12 and 24 mg of selenium per kg of soil in the form of Na_2SeO_4 has revealed a considerable uptake of selenium. But the double dose of selenium fails to double its content in the plants.

Conclusions. 1. Sand cultures are more susceptible than soil cultures to the poisonous influence of selenium. This may be accounted for by the ability of the soil to absorb selenium and convert it into compounds unavailable to the plant.

2. Among the compounds of selenium those with lower degree of oxidation are more poisonous to plants. This conforms to the behaviour of other elements forming compounds of different degrees of oxidation (sulphites and sulphates, nitrites and nitrates).

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MINERALOGY

ALUMINO-CHRYSOTILE, A MEMBER OF THE ISOMORPHOUS SERIES SERPENTINE—PARAKAOLINITE

By D. P. SERDYUCHENKO

(Communicated by D. S. Beliankin, Member of the Academy, 15.V.1944)

In the basin of the Markopidj River, a right-hand tributary of the Malaya Laba at the North Caucasus, along a ravine named by the author the Contact ravine (1936), the present author found in direct contact with unaltered dense serpentinites veined flat accumulations of flattened-columnar green chrysotile. The length of its hard acicular fibres reached 15—20 cm, although not infrequently peculiar breaks and pinchings were obtained.

Along with optical properties normal of chrysotile, this mineral, carefully picked up for analysis under the magnifying lens and the microscope, was found to contain a considerable quantity of alumina and correspondingly, a lower magnesia content.

Table 1

Constituents	Per cent	Molecular numbers	Molecular ratios
SiO_2	42.54	709	2
Al_2O_3	5.68	55	0.17
Fe_2O_3	1.06	7	
FeO	0.74	10	2.52
MgO	35.57	883	
CaO	0.13	—	
$+H_2O$	13.26	742	2.09
$-H_2O$	0.38	—	
	99.36		

Considering the given mineral as an isomorphous mixture ($Mg''_3—Al_2$) of the serpentine and kaolinite (parakaolinite) molecules, we obtain (Table 1) the extreme magnesia member of the isomorphous series (serpentine): $3.03\,MgO \cdot 2SiO_2 \cdot 2.09\,H_2O$; the extreme alumina member of the series (parakaolinite): $1.01\,Al_2O_3 \cdot 2SiO_2 \cdot 2.09H_2O$.

Thus, the mineral investigated has the formula: $H_4(Mg_3,Al_2)Si_2O_9$, and may be named alumino-chrysotile.

The recalculation of the mineral to normative molecules is given in Table 2.

Genetically, alumino-chrysotile is a hydrothermal mineral, associated with a contact action of pre-Cambrian granites upon pre-Cambrian serpentinites. The mineral is located directly at the unaltered serpentinite, being separated from the granite by a zone of talcose, magnetized, chloritized serpentinite, and in close

Table 2

Name of molecules	SiO_2	Al_2O_3	Fe_2O_3	FeO	MgO	H_2O	Total	Per cent	
Antigorite	588	—	—	—	883	588	2059	86.0	87.0
Ferroantigorite	7	—	—	10	—	7	24	1.0	
Kaolinite	110	55	—	—	—	110	275	11.5	13.0
Ferrikaolinite	14	—	7	—	—	14	35	1.5	
	719	55	7	10	883	719	2393	100.0	

proximity to granite, by beautiful skarns. The latter are made up of coarsely crystalline growths of pale greenish-grey or emerald-green amphibole (tremolite and actinolite), monoclinic and rhombic (in serpentinite pseudomorphs) pyroxenes of dark-green colour, dark-brown, almost black (in thin laminae—amber-brown) biotite not infrequently in fine tabular crystals, light-green crystalline-granular epidote, and pink-red vitreous transparent almandine, forming spotted accumulations of skeletal isometric grains.

Long-fibred flattened hard alumino-chrysotile, of similar character, origin and mode of occurrence, was found by the author in 1937 in the same zone of serpentinites and pre-Cambrian granites occurring in contact with the former, on the left bank of the Bolshaya Laba River, upstream of the mouth of the Blyb River along the ravine named by the author the Slyudyanya (with coarsely crystalline amphibole-garnet-biotite skarns).

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PLANT PHYSIOLOGY

ON THE ADAPTATION VALUE OF THE PHOTOPERIODIC REACTION

By A. I. POTAPENKO

(Communicated by A. A. Richter, Member of the Academy, 8.III.1944)

Such universally distributed spring weeds as *Chenopodium album* L., *Amaranthus retroflexus* L., *Polygonum lapathifolium* L., *Bidens tripartitus* L., *Panicum crus galli* L., and some others, even at high latitudes maintain their capacity to respond to a curtailed day by a strong increase in their rate of development. Experiments carried on by the present author in Michurinsk went to demonstrate that if grown at a long-day regime (16—17 hours) these plants are capable to vegetate for about three months. They do, however, break into blossom in this case too, provided their vegetative development has advanced considerably. One succeeds in accelerating their development by gradually reducing the day to less than 17 hours. The highest rate of reproductive development is observed in the case of a 14—13-hour-day regime. A further curtailing of the day down to 12—10 hours does not increase the rate of reproductive development appreciably, while the size of the plants and the amount of seed they yield is found to be markedly reduced. At a 14—10-hour-day regime the plants listed above are capable to develop seed within less than one month.

At certain latitudes the beginning and the middle of the growing period correspond to a long day, while the end of the growing period coincides with a period when the day grows much shorter; under such conditions, owing to the photoperiodic characters here described, the course of the form-building processes ends by being favourably affected in the plants listed above, by the fact that the day grows shorter towards the close of the growing period. Thus, at the latitude of Michurinsk the day-length is on the average 15—16 hours in May, 16 hours in July, whereas in September it is 12—13 hours only. If the plants germinate in spring or early in summer, their reproductive development is protracted; after having formed an abundant vegetative mass with a large leaf area, the plants often yield towards autumn a huge amount of seed. In the case of belated germination the plant gets at once under the conditions of a curtailed day, which is beneficial to its reproductive development; as a result it develops seed at a much higher rate, owing to which both the size of the plants and the yield of seed is reduced.

Certain spring weeds (*Amaranthus retroflexus* L., *Panicum crus galli*, *etc.*) happen to be ravaged by the slightest autumn frosts, which should be put down not so much to the southern origin of those plants, as to their one-sided adaptation to the growing period coinciding with the hot and arid season.

Unlike the tropical short-day plants, spring short-day weeds are badly affected by an excessive reduction of the day-length; their development appears to be inhibited after the latter is brought down to 10 hours. Accordingly, their development proceeds at the highest rate mainly towards the end of the summer and early in

autumn, at a 15—13-hour-day regime. Further reduction leads, as a rule, to the appearance of malformed, dwarfish plantlets which break into blossom without setting any fruit. The inhibitory effect of the autumnal short days is not infrequently aggravated by cloudy weather. Under the conditions which prevail at Michurinsk a well-pronounced development acceleration may be observed as early as during the first days of August (15-hour day); towards the end of August and early in September it attains its maximum, both with respect to the number of plants involved and efficiency (14—13-hour day).

The highest yields of dry matter are obtained from short-day weeds under the conditions of a 17-hour day, whereas tropical plants do so at short-day regime, often with a 7—8-hour day, as has been reported by Lubimenko and Szeglova (1). This should by no means be interpreted as a rudiment of their adaptation to a climate prevailing at low altitudes where these weeds might have originated. On the contrary, it should rather be interpreted as an adaptation of these plants to the specific light conditions which prevail at high latitudes. The short-day habit of spring weeds is a well-pronounced entirely regulatory adaptation. This may be seen, in the first place, from the ability of these plants to break into blossom at a long-day regime after a prolonged period of growth and vigorous vegetative development; their ability to transform into real ephemeras under the conditions of a curtailed day is also evidence of this fact. Moreover, in the case the day is curtailed, blossoms are formed at once instead of the usual lateral shoots in the leaf-sheaths of *A. retroflexus* and *P. lapathifolium* (figure). As a result, the accelerated development of those plants may usually be evaluated in the end of summer and in autumn after their reduced size and some peculiar alterations in their appearance. The recessity to reduce as far as possible the period of growth results in that certain vegetative parts of the plant fail to develop at all.

Procedures aiming at the control of injurious spring weeds use to be suspended by agriculturists in the second half of summer or in the first half of autumn in the hope the very first autumn frosts will have destroyed them before they have time to set seed. They who do so leave out of account, however, that the weeds in question become ephemeras by the end of their growing period.

The response of certain cultivated plants, as for example, millet and hemp, to the day-length is of essentially regulatory character.

In contrast to spring plants, certain hibernating plants such as *Lactuca scariola* L., *Capsella bursa pastoris* Moench. and others, are winter-hardy, as they can go through winter under the snow cover. They germinate chiefly in spring and autumn, when weather is damp and rather cool. The sprouts that peep from under ground in spring get directly under the conditions of a long-day regime beneficial for their rapid flowering; on the contrary, sprouts that appear in autumn are slow to go through the state of the root-rosette. The total produce of photosynthesis is then conveyed to the root and stored there. The plants hibernate in the state of rosette and by the next spring achieve their growing cycle by developing fruit. The expedience of such a course of development assumed by hibernating plants during spring and winter is quite obvious.

All plants may be subdivided into two groups according to their mode of life. In some of them the beginning of vegetation is not adapted to any definite season, and germination may take place throughout the growing period, irrespective of day-length, as in the case of the annuals here described. Nevertheless, they develop certain photoperiodic characters which enable them to adapt the periods of flowering and fruiting to a definite season characterized by a definite day-length.

Others, for example, arboreal plants, are able to adapt their development stages to seasonal weather variations: they work out such annual cycles in which the period of growth alternates with that of rest. The necessity to pass before each growing period a period of rest under the conditions of low temperature makes these plants adapt the beginning of their vegetative development to the beginning of the growing period.

Besides, as shown by Potapenko (2), arboreal plants have worked out a definite period of vegetative development which for any given plant is always shorter than the shortest period of growth, which may possibly exist under the conditions prevailing in the country a given plant originates from. The photoperiodic reaction of arboreal plants is rather dependent on the season with which a given development stage coincides and, consequently, on the day-length at which it has to proceed. As to the coincidence of a given development stage with a definite season, it is achieved without the participation of the photoperiodic reaction.

An adaptation to a definite day-length may possibly be worked out even under the conditions of a rigorously constant length of day and night, *i. e.* irrespective of a possible regulatory effect of the photoperiodic reaction. But if a plant adapted to a definite photoperiod is transferred to conditions of a different photoperiod, its usual course of development is inevitably disturbed thereby. At the same time the adaptation to a definite day-length may serve as a regulatory agent, and acquire the importance of a supplementary adaptation. In so far as there are no such regions all over the globe where the day-length would be constant, the photoperiodic reaction can produce its additional regulatory effect on a rather large scale.

What is then the main and initial adaptation value of the photoperiodic reaction? It should be looked for in the specific influence of the intermittent illumination upon the activity of the assimilatory apparatus of the leaf. For otherwise it can scarcely be conceived why it is the leaf which is an organ susceptible to the effect of photoperiods.

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8. III. 1944.

REFERENCES

[1] V. N. Lubimenko et O. A. Szeglova, C. R., 176 (1923). [2] А. И. Потапенко, Усп. совр. биол., XIII, вып. 1 (1940).

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GENETICS

WAYS OF GENOTYPE STABILIZATION

II. SELECTION OF MODIFYING FACTORS FOR MUTABILITY

By M. V. IGNATIEV and N. I. SHAPIRO

(Communicated by L. A. Orbeli, Member of the Academy, 22.IV.1944)

In our preceding paper [3] we have examined the way of genotype stabilization by selection of the stable allels. This way is not the only possible one.

As has previously been pointed out by us, there is a possibility of selection of modifying factors which reduce mutability; in other words, both individual genes and the genotype as a whole can be stabilized. As has been shown in a number of studies [1, 2, 4], different lines within a species may differ in their mutation rate. In a number of instances these differences have been proved experimentally to be hereditarily determined by special modifying genes for mutability * [1, 5]. The presence of such genes in a population creates the conditions of its dynamics. In the following we briefly report the results of our study devoted to the evolutionary mechanism of the genes-mutators (resp., stabilizers).

1. When a mutator and its allel (a stabilizer) are both present in a population and no reverse mutations occur between them, a full extinction of the mutator should theoretically take place.

2. If reverse mutations occur at the locus of the modifyer, the mutator is preserved at a concentration limited by the mutation rate. In order to reduce the concentration of the gene-mutator, it should possess a very significant strength, *i. e.* cause the production of lethals at a high rate (irrespective of whether one or several loci will mutate).

3. The mechanism of elimination of the mutators consists in the elimination of the «excess» gametes (as compared to the equilibrium gametes) containing both the mutator and the lethal at the same time. «Excessive» combinations of genes are created by the mutation process, and may be preserved throughout a series of generations.

4. The rate of increase in the concentration of the stabilizer for any given moment is equal to the relative number of the «excessive» gametes containing the mutation plus the lethal, dying by the same moment. Thus, the elimination of the mutator proceeds the faster, the greater its strength.

5. The rate of the increase of the concentration of the stabilizers varies as to the plant or animal species according to their chromosomal composition. Three basic types of chromosomal structure of the organism may be pointed out as determining the rate of selection of stabilizers: 1) the haploid, 2) the diploid, and 3) the mixed.**

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* A modifying gene increasing the mutability of other genes will in the following be termed a mutator, while its allel will be called a stabilizer.

** In the adjoined table data are summarized on the rates of the process of elimination of mutators, corresponding to the various types of chromosomal structure of the organism.

Types of evolution	Haploid	Mixed		Diploid	
Localization of recessive mutator		Sex chromosome	Autosome	Autosome	Autosome
Localization of the lethals		» »	Sex chromosome	»	»
Dominance or recessiveness of lethals		Recessive («bee-type»)	Recessive	Dominant	Recessive
Rate of reduction in the concentration of the mutator within a generation	$ua(1-a)$	$\frac{1}{6}ua(1-a)+$ $+\frac{2}{3}ua^2(1-a)$	$\frac{5}{12}ua^2(1-a)$	$\frac{1}{3}ua^2(1-a)$	$2u\sqrt{u}a^2(1-a)$
Duration of the process (in generations) after the first appearance of stabilizer	$\frac{-2.3026\lg_{10}A_0 a}{u}$	$-\frac{6}{u}(2.3026\lg_{10}a+$ $+1.2875+0.460517\lg_{10}A_0)$	$\frac{2.4}{ua}$	$\frac{1}{ua}$	$\frac{1}{4u\sqrt{ua^2}}$
Same with $u=0.001$	16 000	34 000	240 000	100 000	80 000 000
$u=0.01$	1 600	3 400	24 000	10 000	2 500 000
Same with repeated appearance of stabilizer $u=0.001$	4 600	15 000	123 000	70 000	360 000
$v=0.00001$					

6. The process of the elimination of mutators is effected most rapidly in a population of haploid organisms. The duration of the process is determined by an approximate equation *:

$$t = -2.3026 \frac{\lg A_0 a}{u}$$

where t is the number of generations after the first appearance of the stabilizer; a, the ultimate concentration of the mutator; A_0, the initial concentration of the stabilizer; u, the strength of the mutator (relative number of the lethals arising in the population).

7. Into the diploid type belong such diploid organisms as have no chromosomal sex determination (*e. g.*, monoecious plants), as well as all those cases where the mutator is in the autosome and the sphere of its activity is limited to autosomal genes.

8. The rapidity of the elimination of mutators in a population of diploid type varies according to whether the mutator and the lethals produced by it are dominant or recessive.

a) The selection process proceeds most rapidly if the mutator and the lethals are dominant.

b) In the case of intermediacy of the mutator and dominance of the lethals the rate of elimination is somewhat lowered.

c) Still slower proceeds the selection in the case where the mutator is recessive and the lethals are dominant. The duration of the process is expressed in this case by the equation

$$t = 1/ua$$

d) If both the mutator and the lethals are recessive, selection proceeds most slowly. The equation of the duration of the process is

$$t = \frac{1}{4u\sqrt{ua^2}}$$

9. To the mixed type belong such organisms as possess a mechanism of sex-determination (of the XX, XY type, or diplo-haplo) where the mutator or the sphere of its influence (or both) are located in the sex chromosome.

10. The rapidity of elimination of the mutator in the mixed type depends, first, on the specific part played by the haploid type (sex chromosomes); secondly, on the dominance or recessiveness of the mutators and of the lethals produced by them; thirdly, on the positions of the mutator and the lethals in the sex chromosome and the autosomes.

a) The elimination of the mutators proceeds relatively rapidly if both the mutator and the lethals are in the sex chromosome («the bee-type»). On the assumption that both the mutator and the lethal are recessive, the duration of the process will be expressed by the formula

$$t = -\frac{6}{u} (2.3026 \lg_{10} a + 1.2875 + 0.460517 \lg_{10} A_0)$$

The fact that the rapidity of the process in the population of the «bee-type» is composed of two elements is easily explained by the presence in this population of both haploid and diploid organisms.

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* By the duration of the elimination process we understand the number of generations necessary in order that the concentration of the mutator change from the value $1-A_0$ approximating 1 into a value a approximating zero. For the sake of simplicity it is assumed that the stabilizer fully suppresses the mutability of the genome. The concrete values of this and another formulae for different conditions are given in the adjoined table (v is the mutation rate from the mutator to the stabilizer).

b) If the recessive mutator is in the autosome and the recessive lethals are in the sex chromosome, the process proceeds slower. In this case the formula is

$$t = 2.4/ua$$

11. The curve of decline of the mutator concentration in any type is characterized by a low rate of variation at the beginning and at the end of the process, while the movement is a relatively rapid one at the middle. The transition from a moderately high to a moderately low concentrations comprises a relatively small fraction of the whole process.

12. An essential part in the rapidity with which the population is conquered by the stabilizer, belongs to its repeated appearance in the form of recurrent mutations. Re-appearances in many instances produce an effect which approaches to that of the summation of the time-series of mutation rates originating in subsequent generations. Their influence differs in different types, however:

a) In the haploid type the effect of the repeated occurrence of the stabilizer is relatively small and does but little reduce the duration of the process which goes on rapidly. Thus, assuming that the lethal arises at a ratio 1 : 1000 ($u = 0.001$) and the rate of the stabilizer is 1 : 100 000 ($v = 0.00001$), we find that the duration of the process is equal to 4600 generations, *i. e.* about three times less than in the case of a single appearance (16000 generations).

b) In the diploid type where both the mutator and the lethals are recessive, a repeated appearance of the stabilizer has a more important effect. Thus, under the above conditions ($u = 0.001$, $v = 0.00001$) the process is accelerated more than two hundred times.

c) In the mixed type, a repeated appearance similarly accelerates the process of elimination of the mutator. Thus, in the «bee-type» the duration, like in the haploid type, is reduced approximately 2 times. In other cases of the mixed type the process of elimination is accelerated less than twice.

13. In a population of a relatively small size the presence of stochastic processes in a number of cases is likely to cause a more rapid elimination of the mutators than in wide populations. The elimination of the mutators will proceed relatively rapidly also in the case where a large population is segregated into a number of isolated small ones. As an instructive example of the influence of the number of individuals within a population and of the system of crossing may serve a population of self-fertilizing organisms; it may be regarded as a sum of populations each of which consists of a single individual. In such organisms the evolutionary process goes on almost as rapidly as in the haploid type.

14. The beginning of the spread of the stabilizer due to selection must be preceded by a period of its accumulation, which takes from 3 to 40 per cent of the duration of the subsequent process.

15. An important rôle in the acceleration of the preliminary accumulation is played by a repeated appearance of the stabilizer. The importance of the repeated occurrence consists not only in the fact that it accelerates the process, but in that it ensures the success of the stabilizing gene by preserving it from accidental loss.

16. Owing to the repeated occurrence of the stabilizer, the odds for the process of evolution of mutability to take place are about the same in the haploid, mixed and diploid types. The differences in the rate of elimination of the mutators in the types considered become manifest only at later stages, and lead to difference in the duration of the process.

17. Owing to a different rate of the evolution of mutability in different types of populations, organisms differing as to their chromosomal composition may conventionally be arranged in the order of their mutability. The conventionality is due to the assumption that the populations of the organisms have started the evolution of mutability at the same time, and that nothing had interferred with the course of this process.

After a certain time has elapsed, haploid organisms must exhibit the greatest stability. Next follow organisms with a great length of genetically active regions

of the sex chromosomes (assuming the rate of mutation to be proportional to the length of these regions). Organisms in which evolution proceeds according to the diploid type (but not self-fertilized) touch the limit and close the series.

18. The reduction of mutability is one of the ways in which the species reacts to the appearance of lethals. Along with the selection of modifiers of mutability the selection of other modifiers may occur in the population, which, like the reduction of mutability, lead to a reduction of the selective loss suffered by the population. A cumulative effect of a number of processes may consequently be assumed in this case, *viz.* the evolution of the dominance of the lethal and mutator, the evolution of pleiotropy, *etc.* The problem of the relation between the various tendencies raises the study to a rank of the problem of evolution as a whole. The analysis of the evolution of mutability as an isolated process forms an important step toward the solution of this problem.

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6. IV. 1944.

REFERENCES

[1] M. Demerec, Genetics, **22** (1937). [2] N. Giles, ibid., **26** (1941). [3] M. V. Ignatiev and N. I. Shapiro, C. R. Acad. Sci. URSS, XLV, No. 5 (1944). [4] M. Valadares, Rev. Agronom., **25** (1937). [5] Mampell, Proc. Nat. Acad. Sci., **29** (1943).

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ENDOCRINOLOGY

ON THE INTERNAL SECRETION OF INSECTS

By V. V. YAKHONTOV

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At the beginning of the last decade the question as to the existence of internal secretion in insects seemed to be definitely decided in the negative. Numerous experiments made by different research workers who eliminated and transplanted the gonades of insects at any stage of their development, failed to demonstrate any changes in the aspect of the males and females showing a very pronounced sexual dimorphism. In the experiments of Kopeč (4) the transplantation of the wing rudiment of caterpillars from one sex to the other did not affect the sexual characteristics of the developing wing of the butterfly. As the search for hormones not only in insects, but in other invertebrates as well never gave any results but negative ones, R. Goldschmidt proposed to separate the vertebrates into a special group of «hormonal» animals as distinguished from the rest of animals.

Some workers, in particular Kopeč (5) and Buddenbrock (2), in spite of this, surmised the existence of internal secretion in insects. They were inclined to explain the processes of moulting in the nymphs and of pupation by the action of hormones, though they were unable to obtain decisive proofs of the existence of hormonal activity in insects. Several years later the correctness of the assumptions of Kopeč and Buddenbrock was confirmed by the brillant work by Wigglesworth (8) who proved the endocrinal activity of *corpus allatum*, an organ lying in the back of the head beside the sympathic nervous ganglion. The *corpus allatum* of the bug *Rhodnius*, used by Wigglesworth for his experiments, secretes a hormone which is responsible for the moulting. This hormone is produced and secreted into the hemolymph only at a definite period of the bug's nymphal life.

On the ground of his study of a large number of gynandromorphous specimens of *Drosophila simulans* among which there were some with tissues typical of the one sex and with sex organs belonging to the other, Dobrzhansky (3) advanced in the beginning of the past decade the hypothesis that insects possess sexual hormones also. Dobrzhansky, however, did not consider his data sufficiently conclusive for proving the existence of internal secretion in insects. The same assumption might be inferred from the phenomenon of «parasitic castration» found by Salt (7) in some *Hymenoptera*, since the action of their internal parasites, *Stylops*, provoked in some cases the appearance of male characters in infected female *Hymenoptera*, and that of female characters in male ones.

However, this partial change of sex in cases of «parasitic castration» might have been caused also by the deficient diet of infected nymphs, for it was observed in such *Hymenoptera* only which provide their future nymphs with food for the entire period of their nymphal development. It is known, *e. g.*, that the size of the host influences the numeric proportion of males and females in the parasitic *Hymenoptera* o the family *Braconidae*.

Almost simultaneously with these studies Loewe (6) has published the results of

the researches of his laboratory at Goettingen, which proved unquestionably the existence of sexual hormones in insects, in butterflies in particular.

The sex hormones are not specific and all animals equally react to them irrespective of the origin of the hormones, *i. e.* whether they have been taken from animals of the same species, or from animals quite remote in the taxonomic range. Utilizing this principle, Loewe was able to demonstrate the existence of sex hormones in insects by injecting the blood of butterflies subcutaneously to mice. Yet even after the presence of sex hormones in insects had definitely been proved, the rôle of these hormones in insect physiology remained as obscure as before.

A most interesting study was reported recently by Mokiya (1), showing the influence of the sex hormones of the mulberry silk-worm on the rate and character of its oviposition.

In the course of one of our studies we have also encountered an illustrative, as it seems to us, case of the effect of sex hormones on the life of insects. Two species of the family *Coccinellidae—Brumus octosignatus* Gebl. and *Semiadalis undecimnotata* Schneid.—in Uzbekistan and Southern Kazakhstan hibernate in large accumulations on the summits of mountains. In the spring, with the appearance of aphides on the xerophilic plants, the beetles fly out from their accumulations to the valleys. This exodus from the mountains proceeds gradually not only depending on the difference in height, but even from the same hibernation sites. It was found that neither temperature nor humidity, or any other meteorological factor do exert any effect upon the date and rate of the outflying of the *Coccinellidae* to the valleys. Neither are these influenced by the diminution of the fat supplies stored in the body of the hibernating beetles. Only those beetles fly out of the winter sites, in which the sex products have begun to mature. Dissecting roughly the seminal glands of the males at this stage, it is easy to observe that they have become yellow, the glands begin to excrete mature spermatozoa, in the ovaria of the females the inferior egg chamber is formed. Copulation precedes outfly of insects.

It is evident that the hormonal activity of the sex glands produces in the migratory *Coccinellidae* the stimulus towards the development of the flying instinct, similarly to what has been demonstrated in the past decade in respect of birds by Schildmacher at the Helgoland Ornithological Station. Meteorological conditions (temperature in particular) influence the exodus of the insects indirectly by determining the time of ripening of their sex products. The beginning of the maturing of sex products in the *Coccinellidae* depends also on the sudden fall in their fat stock.

It may be surmised that the same part is played by sex hormones in some other migratory insects, in locusts (*Locusta migratoria* L.), *e. g.*, as well as in other gregarious species of the family *Acrididae*, the cause of the migrations of which remain a riddle up to the present, though the locust is one of insects whose biology is most thoroughly studied. At any case the beginning of the migration of the locusts, like that of the *Coccinellidae*, coincides with the beginning of the maturing of their sex products.

At the present time the question as to the existence of internal secretion in insects is evidently solved in an opposite sense to the solution accepted by the beginning of the 30-ies of our century, although the secondary sex characters of the insects, at least in the great majority of cases, do not actually depend on their endocrine activities. Internal secretion does not only exist in insects, but it shows a great functional diversity (hormones of moulting, hormones affecting the rate of oviposition, migration stimulating hormones) and is bound with various glands (additional body in the cephalic nervous mass, sexual system).

Received
28. II. 1944.

REFERENCES

[1] G. G. Mokiya, C. R. Acad. Sci. URSS, XXX, No. 4 (1941). [2] W. Buddenbrock, Z. Morph. Oekol. Tiere, XVIII, 700 (1930); Z. vergl. Physiol., XIV, 415 (1931). [3] T. Dobrzhansky, Roux'Archiv-f. Entwickl. d. Org., 123, H. 3/4, 719 (1931). [4] S. Kopeč, J. exp. Zool., XXXVI, 459 (1922). [5] S. Kopeč, Biol. Bull., XLVI, 1 (1924). [6] S. Loewe, Naturwiss., H. 37 (1931). [7] G. Salt, J. exp. Zool., XLVIII, 223 (1927); ibid., LIX, 133 (1931). [8] V. Wigglesworth, Quart. J. Micr. Sci., LXXVII (1934).

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